Math/CS 103 Professor: Padraic Bartlett

Minilecture 5: Projective Planes

Week 5 *UCSB 2014*

1 Projective Planes

Definition. A **projective plane** is a collection of points and lines in space that follow the following fairly sensical rules:

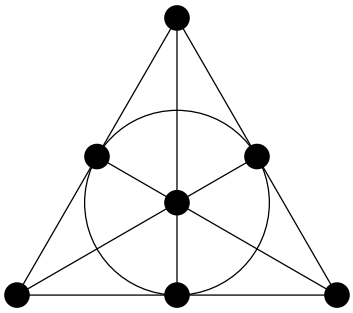
(P1): Given any two points, there is a unique line joining any two points.

(P2): Any two distinct lines intersect at a unique point.

(P3): There are four points, no three of which are collinear.

Basically, these are the affine plane axioms, except we removed the ''Given any line L and point P, there is exactly one line parallel to L through P" property, and replaced it with the axiom "There are no parallel lines."

Here is an example of a projective plane containing seven points and seven lines:



The seven lines above are the three faces of the triangle, the three bisectors through the center of the triangle, and the circle (which is a single line.)

Projective planes are intimately related to affine planes, as you will show on the HW:

Proposition. Take any projective plane P. Pick a line in P, and delete that line along with all of the points on that line. The resulting set of points and lines is an affine plane.

Proposition. Take any affine plane A. Divide A's lines into $n+1$ parallel classes $C_1, \dots C_{n+1}$. For each class C_i, add a point ∞_i to our plane, and have every line of C_i go through ∞_i. Finally, add a line consisting of all of the points $\infty_1, \dots \infty_{n+1}$.

This creates a projective plane.