CCS Discrete Math I — Professor: Padraic Bartlett

Homework 1: Basic Counting

Due Friday, Week 1 — *UCSB 2014*

Do **three** of the **five** problems below! Prove all of your claims. If you've seen some of these problems before, try the ones you have not seen first.

1. Show that the following equality holds for all $n \in \mathbb{N}$:

$$2^n = \sum_{i=0}^{n} \binom{n}{i}.$$

2. In class, we showed that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

by creating a $(n+1) \times (n+1)$ square out of dots and counting in two ways.

Using similar methods, try to count $\sum_{i=1}^{n} i^2$.

3. **Without** using induction, show that the following equality holds for all $a, b, n \in \mathbb{N}$:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

4. Show that the following equality holds for all $n \in \mathbb{N}$:

$$\sum_{i=1}^{n} i \cdot (n-i) = \sum_{i=1}^{n} \binom{i}{2} = \binom{n+1}{3}.$$

5. Call an ordered list of the first n numbers $[n] = \{1, 2, 3, \ldots n\}$ **pleasant**[1] if it satisfies the following property:

   - The first element in our list can be any of the numbers in $\{1, 2, 3, \ldots n\}$.
   - If k is any element in our list that is not the first element, then at least one of $k+1, k-1$ must have shown up earlier in our list.

   For example,

$$(2, 3, 4, 5, 6, 1)$$

   is a pleasantly ordered list. However,

$$(2, 3, 5, 4, 6, 1)$$

   is not a pleasantly ordered list: we have 5 in the third position, but we do not have either 4 or 6 occurring earlier in our list.

   How many ordered lists are there of $[n]$?

[1] I made this term up.