CCS Discrete Math I — Professor: Padraic Bartlett

Homework 8: The Symmetric Group

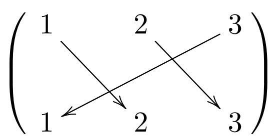
Due Friday, Week 4 — *UCSB 2014*

This problem set is different. Do the **one** problem on this set! (It's an important enough problem that I want you to do just this one question, instead of the typical $\binom{n}{k}$-style we've used so far.)

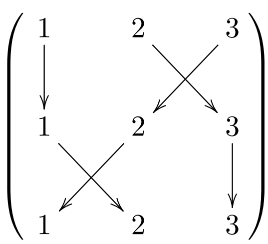
1. In class, we showed that you can write any element of S_n as a product of **transpositions**: that is, permutations of the form (ab) for distinct $a, b \in \{1, \dots n\}$. However, there are many ways to write the same permutatuon $\sigma \in S_n$ as a product of transpositions: for example, we have

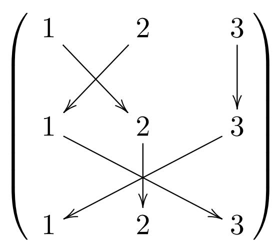
$$(123) = (13)(12) \text{ and } (123) = (12)(23),$$

because

(123) = 

is the same as both

(12)(23) = 

and (13)(12) = 

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Suppose that $\sigma \in S_n$ is a permutation that can be written as a product of an **odd** number of transpositions. Prove that it cannot also be written as a product of a **even** number of transpositions.

(In this sense, the **parity** of the number of transpositions used to create any permutation $\sigma \in S_n$ is an **invariant** for that permutation.)