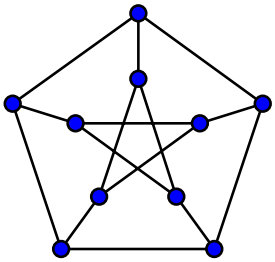
CCS Discrete III | Professor: Padraic Bartlett

Homework 1: Circuits and Random Walks

Due Friday, Week 1 | *UCSB 2015*

Do **one** of the following **three** problems!

1. Consider the Petersen graph:



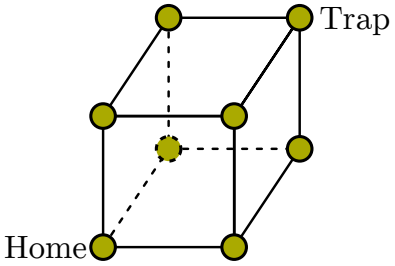
Turn the Petersen graph into a circuit as follows: Pick any two nonadjacent vertices S, G in this graph. Set one to be the source and the other to be grounded. Define the voltage at the source to be 1, the voltage at ground to be 0, and the resistance of every edge of this graph to be 1.

   (a) What is the effective resistance of this circuit?

   (b) Did it matter what vertices you chose for S, G? Or is the answer the same for any two such vertices?

2. Take a cube.

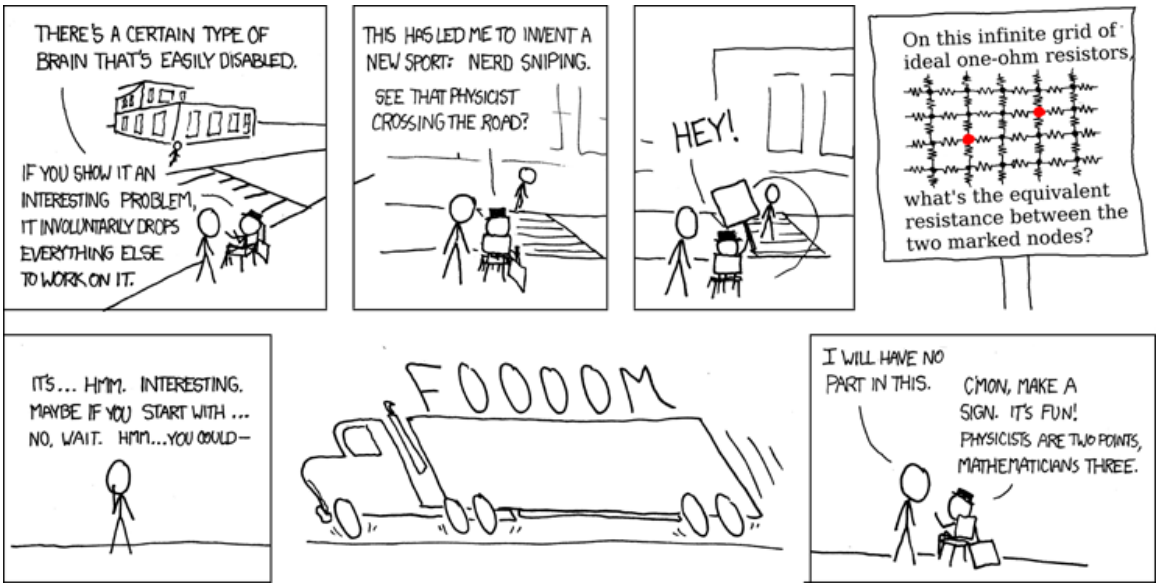
   (a) Suppose that we have an ant walking randomly on the edges of this cube, that starts at one corner and wanders until it either returns to that corner, or makes it to the opposite corner.



   What are the odds that the ant makes it to the opposite corner?

   (b) Generalize the problem above as follows: take a n-dimensional cube, which you can think of as all of the points in $\mathbb{R}^n$ with coordinates of the form $(\pm 1, \pm 1, \dots \pm 1)$. Suppose that a random walker starts at $(-1, -1, -1, \dots -1)$, and randomly walks along edges of this cube until they either return to $(-1, -1, -1, \dots -1)$ or make it to $(1, 1, 1 \dots 1)$. What are the chances that the walker makes it to $(1, 1, 1 \dots 1)$?

3. Solve the problem in this XKCD cartoon:



https://xkcd.com/356/

(Warning: hard!)