CCS Discrete III — Professor: Padraic Bartlett

Homework 13: Linear Algebra

Due Friday, Week 7 — *UCSB 2015*

Solve **one** of the following **three** problems!

1. Prove the following claims about permutation matrices:
   (a) If P is a permutation matrix with associated permutation σ, then P^{-1} is a permutation matrix with associated permutation σ^{-1}.
   (b) If P is a $n \times n$ permutation matrix with associated permutation σ, then $(v_1, \ldots v_n) \cdot P = (v_{\sigma^{-1}(1)}, \ldots v_{\sigma^{-1}(n)})$.

2. Consider the Pell sequence $\{p_i\}_{i=1}^{\infty}$, defined recursively as follows:
   - $p_0 = 0$.
   - $p_1 = 1$.
   - $p_n = 2p_{n-1} + p_{n-2}$.

   The first ten Pell numbers are listed here:

   $$0, 1, 2, 5, 12, 29, 70, 169, 408, 985, \ldots$$

   (a) Find a matrix $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that

   $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} p_n \\ p_{n-1} \end{bmatrix} = \begin{bmatrix} p_{n+1} \\ p_n \end{bmatrix}.$$

   (b) Find the eigenvalues and corresponding eigenvectors of P.
   (c) Use this information along with the methods we discussed in class to find p_{50}.

3. Consider the following two-player game that you can play on a $n \times n$ grid:
   - There are two players, 1 and 2.
   - These two players alternate putting real numbers into the entries of the matrix.
   - Once the matrix is filled, player 1 wins if the determinant of this matrix is nonzero; player 2 wins if the determinant is zero.

   Can you find a strategy for one of these players that guarantees that they'll win?