CCS Discrete III | Professor: Padraic Bartlett

Homework 14: Determinants

Due Friday, Week 8 | *UCSB 2015*

The determinant has many definitions and many properties/applications. We listed four definitions in class on Wednesday, and gave many different properties of the determinant. In this set, you're asked to prove many of these properties! Specifically, consider the following list:

1. Any two of the four definitions of the determinant are equivalent.
2. For any two $n \times n$ matrices A, B, $\det(AB) = \det(A)\det(B)$.
3. For any matrix A, if A^T denotes the **transpose** of A (i.e. $A^T(i,j) = A(j,i)$), then $\det(A^T) = \det(A)$.
4. For any matrix A, the determinant is zero if and only if the rows of A are linearly dependent if and only if the columns of A are linearly dependent.
5. For any matrix A, λ is an eigenvalue of A if and only if $A - \lambda I$ has determinant 0. (Notice that $\det(A - \lambda I)$ is actually a polynomial in λ, if we think of all of the entries in A as constants: we call this the **characteristic polynomial** of A.)
6. For any matrix A, the determinant is unchanged if we add multiples of one row in A to another row in A, gets multiplied by -1 if we swap two rows in A, and gets multipled by λ if we scale any row by λ. (The same holds for columns.)
7. If G is any undirected finite graph on n vertices, then the spectrum of G consists of n values (counted with multiplicity.)
8. Suppose that a $n \times n$ matrix A has n eigenvalues, counted with multiplicity. Then the determinant of A is the product of its eigenvalues.

Choose **four** of the above to prove! (You may choose problem 1 up to three times; i.e. each successful completion of problem 1 consists of choosing a pair of definitions and showing that they are equivalent. If you have already shown that $A \Leftrightarrow B$ and $B \Leftrightarrow C$, you cannot choose $A \Leftrightarrow C$ for a third problem to solve. When solving any non-1 problem, you may assume that the four definitions from class are all equivalent; i.e. you can use whichever definition you prefer.)

You will likely want to carefully read the notes and perhaps consult outside sources/talk to me in class/OH!