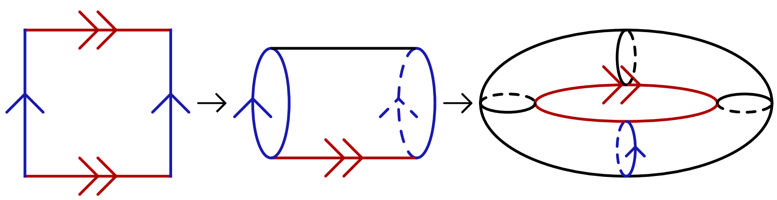
CCS Discrete III	Professor: Padraic Bartlett

Homework 16: Toroidal Graphs

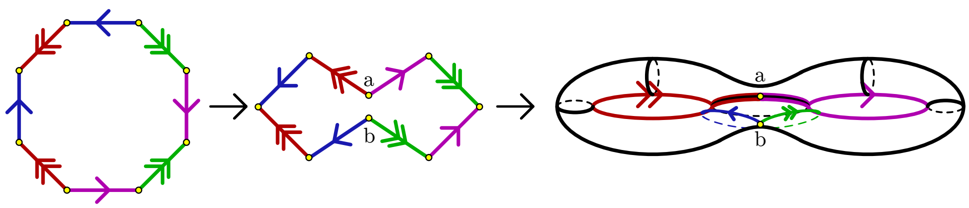
Due Friday, week 10 — *UCSB 2015*

Pick **three** of the problems in this set to solve! Solutions need justification and proof to receive full credit: i.e. it is not enough to simply draw the answer.

1. In class, we created a way to glue together sides of a square to make a torus:



The picture below shows how to glue together sides of an octagon to create a "two-hole torus:"



(a) Create a way to glue together the sides of a square to get a sphere.

(b) Create a way to glue together the sides of a $4n$-gon to get a n-hole torus.

(c) Create a way to glue together the opposite sides of a hexagon to get a torus.

2. A graph G drawn on a n-hole torus is called n-**toroidal** if it satisfies the same definition we gave in class (i.e. we can draw it on a n-hole torus so that no edges intersect and the regions bounded by edges look like open regions of $\mathbb{R}^2$.)

(a) Prove that if G is a 2-toroidal graph, then $V - E + F = -2$.

(b) Generalize the above problem: show that if G is an n-toroidal graph, then $V - E + F = 2 - 2n$.

3. Suppose that G is planar. Prove that there is a planar embedding of G in the plane where all of the edges are drawn with straight line segments.

4. (a) Show that there is no connected bipartite 3-regular planar graph on 10 vertices.

(b) Show that for any even $n \geq 4, n \neq 5$, there is a connected bipartite 3-regular planar graph on $2n$ vertices.

5. In class we proved Heawood's formula, which in the case where $g = 0$ is the four-color theorem! Explain why our proof fails for $g = 0$.