CCS Discrete III — Professor: Padraic Bartlett

Homework 9: Algebraic Flows

Due Friday, Week 5 — *UCSB 2015*

Do **one** of the following **three** problems! Have fun!

1. Show that $\varphi(\text{Pete}) = 5$, by proving that no 4-flow exists on the Petersen graph, while a 5-flow does exist.

2. Let G be a multigraph containing an edge $e = \{x, y\}$, where $x \neq y$. Let G/e denote the graph formed by "contracting" the edge e to a single point. Finally, let A be any abelian group.

   Prove, as claimed in class, that there is a 1-1 correspondence between A-flows on G/e and A-circulations on G that are nonzero on every edge except for maybe e.

3. In class, we claimed that a graph has a k-flow if and only if it has a $\mathbb{Z}/k\mathbb{Z}$-flow. Prove this!