CCS Problem-Solving I — Professor: Padraic Bartlett

Homework 13: Filling Space

Due Thursday, Week 7, at the start of class. — *UCSB 2014*

These are three problems I once received in the same set in undergraduate; they're three of my favorite problems, and I've been waiting all quarter for the ability to give you them as a set.

Solve **one** of the following **three** problems. As always, prove your claims! (And have fun!)

0. Solve any un-signed-up-for problems from HW#12!

1. A **t-shape** is any pair of straight closed line segments in $\mathbb{R}^2$ that intersect at exactly one point, with this intersection not occurring at an endpoint of one of these line segments.

   A **covering** of $\mathbb{R}^2$ by disjoint t-shapes is any collection of t-shapes such that every point of $\mathbb{R}^2$ is in exactly one t-shape.

   Can such a covering exist?

2. A **circle** in $\mathbb{R}^2$ is exactly what you think it is: the collection of all points that are exactly distance r away from some point (a, b) in space, for some value $r > 0$ and point (a, b). Note that r needs to be strictly positive, as a single point is not typically considered as a "circle."

   Is it possible to create a covering of $\mathbb{R}^2$ by disjoint circles?

3. The concepts of circles and coverings both make sense in $\mathbb{R}^3$ as well. A circle in $\mathbb{R}^3$ given by a point (a, b, c), a plane passing through this point, and a distance $r > 0$ is just all of the points that are distance r from (a, b, c) and contained within our plane. Again, note that r needs to be strictly positive, as a single point is not typically considered as a "circle."

   Similarly, a covering of $\mathbb{R}^3$ by disjoint circles is just any collection of disjoint circles such that every point in $\mathbb{R}^3$ is in exactly one circle.

   Can such a covering exist?