Knot Theory
Presentation 1

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- History of Knot Theory: Knots have been in use since the invention of rope around 4000 BC. Knots were also regarded as having religious and spiritual symbolism in many ancient religions. Mathematical Theory of knots was developed first in 1771 by Alexandre-Thophile Vandermonde, a french musician, chemist and mathematician who saw topological importance in the properties of mathematical knots. The actual study of knot theory began with Gauss who developed an invariant called the linking number and the linking integral for computing said linking number. (We will teach about these in further lectures).

  Knot Theory has many applications in topology, biology, chemistry and physics with the most notable application being the unknotting of DNA.

  (South Alabama University)

- Knot Definition: A knot is defined as a circle embedded in $\mathbb{R}^3$. May also be defined as a continuous mapping $f : [0,1] \to \mathbb{R}^3$ such that $f(0) = f(1)$ and $f(x) = f(y) \implies x = y$, $x = 0$ and $y = 1$, or $x = 1$ and $y = 0$

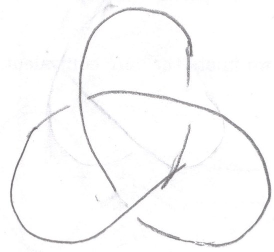
  (Stanford)

- Mentionable Knots:

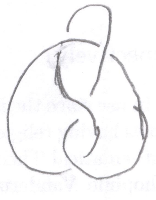
  1. The Unknot (0)

  2. The Trefoil (3_1)

3. Figure-eight knot (4_1)



4. "Carrick Mat" (8_{18})

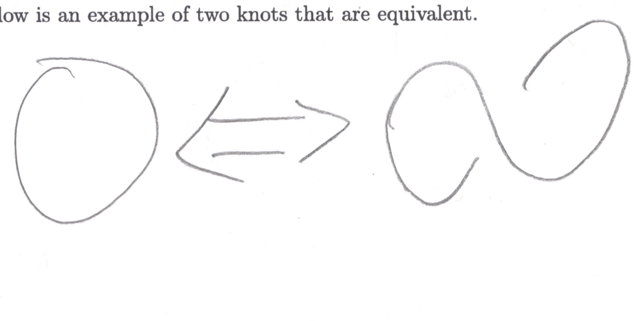
- Regular Projection: A regular projection is how we show knots on the board. Due to the problem of crossing ambiguity, the knot must be oriented in a way such that no three points on the knot project to the same point. A projection of a knot with the previous property defines a "regular" projection.

  A crossing is defined as when a point on the knot passes by another. Crossings are often used to characterize knots.

- Knot Table: In order to distinguish between knots, there exists a knot table that assigns different types of knots to a natural number subscripted with another to each knot of 11 or less crossings. Knots are assigned a_b where a denotes the number of crossings and b denotes it's number on the list of knots with a crossings.

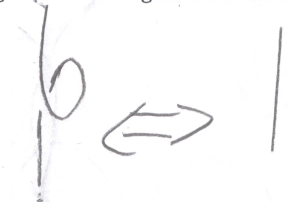
- Knot Equivalence: Two knots are equivalent when one can be transformed into the other, without having to intersect or cross with itself. Knot equivalence is an equivalence relation. (Wikipedia - Adam 2004)

Below is an example of two knots that are equivalent.

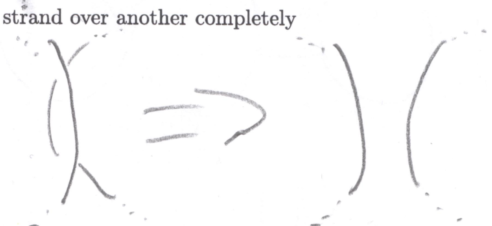


- Reidemeister Moves: Reidemeister, formulated by J. W. Alexander and G. B. Briggs, and Kurt Reidemeister independently, are a sequence of three moves in which two knot diagrams belonging to the same knot can be related. (Wikipedia - Adam 2004) The Reidemeister are the following:

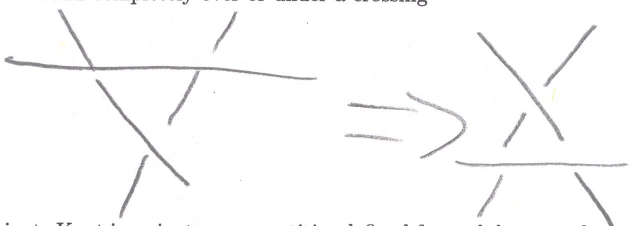
  1. Twisting and untwisting in either direction



  2. Move one strand over another completely



  3. Move a strand completely over or under a crossing



Knot Invariant: Knot invariants are quantities defined for each knot, such that every equivalent knot has the same knot invariant. However, different knots may have the same invariant, and thus, the knot invariants are unable to determine all possible knots. (wolfram mathworld)

- Knot Addition: Two knots can be added by cutting both knots and linking the ends from one knot to the other. A formal definition is as follows. Consider two knots that are disjoint projected in a plane. Now, find a rectangle in which two opposite sides are part of each knot. Now, delete the two sides of the rectangle that are part of the knot, and keep the other two sides. The resulting knot is the sum of the first two knots. (Wikipedia - Adam 2004) Below is an example of two knots being added.