CCS Problem-Solving II | Professor: Padraic Bartlett

Project 1: Latin Squares

Weeks 1-10 | *UCSB 2015*

An order-n **Latin square** is a $n \times n$ array filled with the symbols $\{1, 2, \ldots n\}$, such that no symbol is repeated in any row or column. Here are a few Latin squares of small orders:

2	1
1	2

,

1	2	3
2	3	1
3	1	2

,

1	4	3	2
2	3	1	4
4	1	2	3
3	2	4	1

.

A **partial Latin square** is a $n \times n$ array in which cells can either be blank or contain a symbol $\{1, \ldots, n\}$, such that no symbol occurs twice in any row or column. A Latin square L is called a **completion** of a partial Latin square P if L and P agree on all of P's nonblank entries.

Some partial Latin squares have completions:

$P =$

	2	
3	1	2

$\longmapsto L =$

1	2	3
3	1	2
2	3	1

Others do not:

$P =$

1		
	1	
		2

has no completion.

Determining what kinds of partial Latin squares have completions is an open question in many situations! Here are some known results:

- (Hall, 1949:) A **Latin rectangle** is a partial Latin square P where the first k rows of P are filled and the rest are blank. All Latin rectangles can be completed.
- (Smetaniuk, 1981:) If P is a partial latin square with $\leq n - 1$ filled cells, P can be completed.
- (Buchanan, 2007:) If P is a $n \times n$ partial Latin square where precisely 2 rows and columns of P are filled, P can be completed.

A question I spent part of my dissertation studying is the following: Call a $n \times n$ partial Latin square ϵ**-sparse** if at most ϵn of the entries in any row, column, or symbol are not blank. It is conjectured that all 1/4-sparse partial Latin squares are completable; in my dissertation I proved that all 10^{-4}-dense partial Latin squares are completable.

This result is improvable, in many different ways. Students on this project will attempt to find such improvements!