Speed Limits in “Slife”

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Last week we introduced “The Game of Slife”, and talked about some things we had learned. Let us begin with a summary of what we know, and what we still wanted to figure out.

Things We Know:

- Still Lives in The Game of Life are also still lives in The Game of Slife.

  The proof of this was very simple. In a still life, we never have to update cells. So, it does not matter whether we update simultaneously or sequentially, because there is nothing to update.

- There are spaceships in The Game of Slife.

  We know this because we found two of them.

Things We Did Not Know:

- Are there oscillators in Slife?
- Are there more spaceships in Slife?
- Are there any directions that spaceships cannot travel in?
- What are the speed limits for Slife Spaceships?

Last week we began formulating thoughts that may answer our question regarding speed limits, but we could not come up with anything concrete. However, this week we have been developing our arguments, and will present the results we have discovered.

These results all have to do with speed limits of spaceships moving in specific directions, and there are two types of motion that we have studied: orthogonal and diagonal. First we will discuss the orthogonal speed limits.

As a reminder, c, the speed of light in “Life” is 1 cell per generation. Because there is an update order, it seems plausible that some things may actually be able to move faster than that. The question of “how much faster?” is something that we can answer.

Orthogonal Speed Limits:

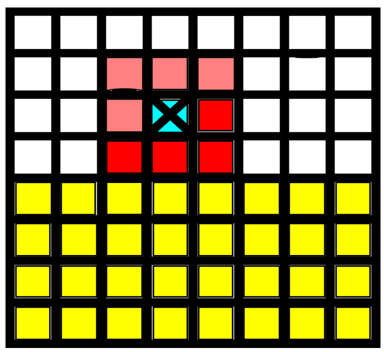
For a spaceship to be traveling **orthogonally**, it must be traveling in one of the following directions: positive y, negative y, positive x, or negative x. We may also refer to these as north, south, east, or west, respectively.

So for example, one could ask the question: "What is the speed limit for a spaceship travelling north?"

Claim 1: That the north speed limit is less than or equal to 1 cell per generation, c.

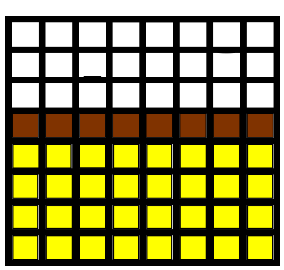
Proof. By Contradiction.

Assume we could move 2 cells per generation. Then, look at the following grid which we could play Slife on.



Assume that any of the yellow cells could possibly be alive at time $t = 0$, and assume that that the blue cell with the X through it is alive at time $t = 1$. This means that exactly 3 of the pink and red cells need to be alive before updating the blue cell. Because the update order is "left to right, top to bottom," none of the red cells can be alive in time. Furthermore, all of the pink cells are too far away from the yellow cells to have any neighbors and come to life in time to make the blue cell come to life. Therefore, the blue cell cannot be alive at time $t = 1$.

This implies that the only cells that could be alive at time $t = 1$ are the brown cells in the following grid.



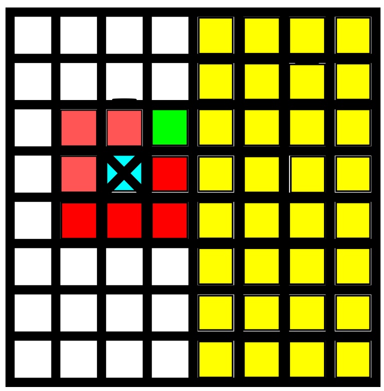
This situation is just like above, and we know that again we can only move 1 more cell by time $t = 2$. In general this tells us that it takes at least n generations to move north n cells. So the north speed limit is less than or equal to c. □

So, we have proven an upper bound on the northern speed limit, but let's see what other speed limits we can find.

Claim 2: The western speed limit is less than or equal to 1.

Proof. By Contradiction.

Assume we could move 2 cells per generation. Then, look at the following grid which we could play Slife on.

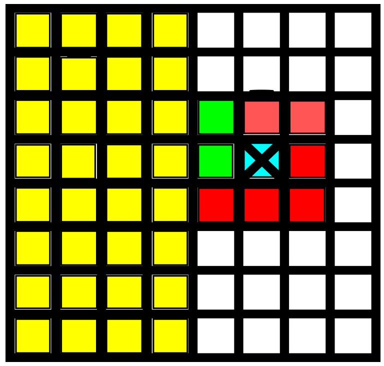


Again, assume that any of the yellow cells could possibly be alive at time $t = 0$, and assume that the blue cell with the X through it is the furthest north living cell that is distance 2 from the yellow cells at time $t = 1$. None of the red cells can be alive by the time we need to update the blue cell. None of the pink cells can be alive because they are too far away. So, the green cell is the only one that could be alive is the green cell. This means that the blue cell will only have at most one neighbor and cannot come to life by time $t = 1$. By a similar argument that proved Claim 1, it follows that the western speed limit is less than or equal to 1. □

Claim 3: The eastern speed limit is less than or equal to c.

Proof. By Contradiction.
Almost exactly like the preceding proof, if the blue cell is the furthest north live cell that is 2 cells away from the yellow cells, the red cells will not be alive in time and the pink cells are too far away. This leaves only 2 possible neighbors for the blue cell, and the eastern speed limit is at most c. □

This concludes the orthogonal speed limits that we know how to bound. We have not yet come up with a good bound on a southern speed limit. Now we will explore diagonal speed limits.



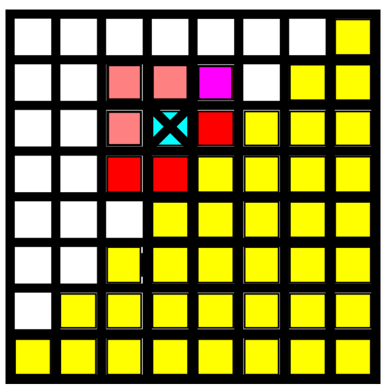
Diagonal Speed Limits:

For our purposes, **diagonal** motion refers to cells that travel n cells in the $\pm x$ direction for n cells in the $\pm y$ direction.

Claim 4: The north-west speed limit is at most $c/2$.

Proof. By Contradiction.

Look at the following grid.

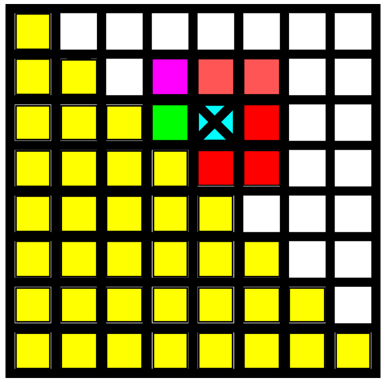


Let the blue cell be the furthest north cell off of the diagonal at time $t = 1$. The red cells cannot be alive in time to bring the blue cell to life. The pink cells are too far away to come to life before updating the blue cell, and the purple cell is further north than the blue cell, so we would have chosen it if it were alive. So, the blue cell does not have enough neighbors to come to life by time $t = 1$. Again, by the argument made in Claim 1, it will take at least 2 generations to move 1 cell in the north-west direction, so the speed limit is at most $c/2$. □

Claim 5: The north-east speed limit is at most $c/2$.

Proof. By Contradiction.

Look at the following grid.

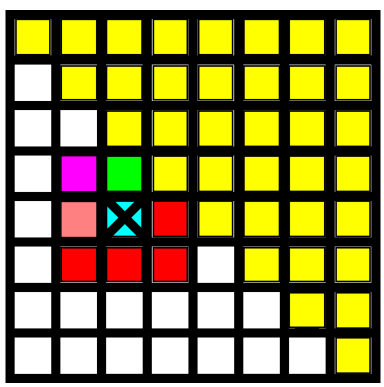


As before, let the blue cell be the furthest north cell off of the diagonal at time $t = 1$. The red cells cannot be alive yet, the pink cells are too far away to be alive, and if the purple cell were alive we would have chosen it. The green cell could be alive, but the blue cell still only has 2 neighbors. Therefore, the blue cell cannot be alive at time $t = 1$ and the north-east speed limit is at most $c/2$. □

Claim 6: The south-west speed limit is at most $c/2$.

Proof. By Contradiction.

Look at the following grid.



Let the blue cell be the furthest north cell off of the diagonal that is alive at time $t = 1$. The red cells will not be alive in time to bring the blue cell to life. The pink cell is too far away to be alive. The purple cell is too far north. The green cell could be alive, but the blue cell only has at most 2 neighbors, so it cannot come to life by time $t = 1$. Therefore, the southwest speed limit is at most $c/2$. □

This concludes the diagonal speed limits that we know how to find bounds for.

We would like to know some bounds for the speed limits in the directions that we have not yet found, and are making progress in finding these bounds, but have not yet been able to formalize our arguments.