FORM GR9367

67

THE GRADUATE RECORD EXAMINATIONS®

GRE®

MATHEMATICS TEST

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MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

Note: In this examination:

(1) All logarithms are to the base e unless otherwise specified.
(2) The set of all x such that $a \le x \le b$ is denoted by $[a, b]$.

1. If $f(g(x)) = 5$ and $f(x) = x + 3$ for all real x, then $g(x) =$

(A) $x - 3$ (B) $3 - x$ (C) $\frac{5}{x+3}$ (D) 2 (E) 8

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2. $\lim_{x \to 0} \frac{\tan x}{\cos x} =$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) $+\infty$

---

3. $\int_0^{\log 4} e^{2x} dx =$

(A) $\frac{15}{2}$ (B) 8 (C) $\frac{17}{2}$ (D) $\frac{\log 16}{2} - 1$ (E) $\log 4 - \frac{1}{2}$

---

4. Let $A - B$ denote $\{x \in A : x \notin B\}$. If $(A - B) \cup B = A$, which of the following must be true?

(A) B is empty
(B) $A \subseteq B$
(C) $B \subseteq A$
(D) $(B - A) \cup A = B$
(E) None of the above

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5. If $f(x) = |x| + 3x^2$ for all real x, then $f'(-1)$ is

(A) -7 (B) -5 (C) 5 (D) 7 (E) nonexistent

---

6. For what value of b is the value of $\int_b^{b+1} (x^2 + x)\,dx$ a minimum?

(A) 0 (B) -1 (C) -2 (D) -3 (E) -4

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7. In how many of the eight standard octants of xyz-space does the graph of $z = e^{x+y}$ appear?

(A) One (B) Two (C) Three (D) Four (E) Eight

---

8. Suppose that the function f is defined on an interval by the formula $f(x) = \sqrt{\tan^2 x - 1}$. If f is continuous, which of the following intervals could be its domain?

(A) $\left(\frac{3\pi}{4}, \pi\right)$

(B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(D) $\left(-\frac{\pi}{4}, 0\right)$

(E) $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$

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9. $\int_0^1 \frac{x}{2 - x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{5}{3}$ (C) $\frac{\log 2 - e}{2}$ (D) $-\frac{\log 2}{2}$ (E) $\frac{\log 2}{2}$

---

10. If $f''(x) = f'(x)$ for all real x, and if $f(0) = 0$ and $f'(0) = -1$, then $f(x) =$

(A) $1 - e^x$ (B) $e^x - 1$ (C) $e^{-x} - 1$ (D) e^{-x} (E) $-e^x$

---

11. If $\phi(x, y, z) = x^2 + 2xy + xz^{\frac{3}{2}}$, which of the following partial derivatives are identically zero?

I. $\frac{\partial^2 \phi}{\partial y^2}$

II. $\frac{\partial^2 \phi}{\partial x \partial y}$

III. $\frac{\partial^2 \phi}{\partial z \partial y}$

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

GO ON TO THE NEXT PAGE.

12. $\lim_{x \to 0} \frac{\sin 2x}{(1 + x)\log(1 + x)} =$

(A) -2 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

---

13. $\lim_{n \to \infty} \int_1^n \frac{1}{x^n}\, dx =$

(A) 0 (B) 1 (C) e (D) π (E) $+\infty$

---

14. At a 15 percent annual inflation rate, the value of the dollar would decrease by approximately one-half every 5 years. At this inflation rate, in approximately how many years would the dollar be worth $\frac{1}{1{,}000{,}000}$ of its present value?

(A) 25 (B) 50 (C) 75 (D) 100 (E) 125

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15. Let $f(x) = \int_1^x \frac{1}{1+t^2}\,dt$ for all real x. An equation of the line tangent to the graph of f at the point $(2, f(2))$ is

(A) $y - 1 = \frac{1}{5}(x - 2)$ (B) $y - \text{Arctan } 2 = \frac{1}{5}(x - 2)$ (C) $y - 1 = (\text{Arctan } 2)(x - 2)$

(D) $y - \text{Arctan } 2 + \frac{\pi}{4} = \frac{1}{5}(x - 2)$ (E) $y - \frac{\pi}{2} = (\text{Arctan } 2)(x - 2)$

---

16. Let $f(x) = e^{g(x)}h(x)$ and $h'(x) = -g'(x)h(x)$ for all real x. Which of the following must be true?

(A) f is a constant function.
(B) f is a linear nonconstant function.
(C) g is a constant function.
(D) g is a linear nonconstant function.
(E) None of the above

---

17. $1 - \sin^2\left(\text{Arccos } \frac{\pi}{12}\right) =$

(A) $\sqrt{\dfrac{1 - \cos\frac{\pi}{24}}{2}}$ (B) $\sqrt{\dfrac{1 - \cos\frac{\pi}{6}}{2}}$ (C) $\sqrt{\dfrac{1 + \cos\frac{\pi}{24}}{2}}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi^2}{144}$

GO ON TO THE NEXT PAGE.

18. If $f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}$ for all $x \in (0, 1)$, then $f'(x) =$

(A) $\sin x$ (B) $\cos x$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-2x}{(1+x^2)^2}$ (E) $\frac{2x}{(1-2x)^2}$

19. Which of the following is the general solution of the differential equation

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = 0\ ?$$

(A) $c_1e^t + c_2te^t + c_3t^2e^t$

(B) $c_1e^{-t} + c_2te^{-t} + c_3t^2e^{-t}$

(C) $c_1e^t - c_2e^{-t} + c_3te^{t^2}$

(D) $c_1e^t + c_2e^{2t} + c_3e^{3t}$

(E) $c_1e^{2t} + c_2te^{-2t}$

GO ON TO THE NEXT PAGE.

20. Which of the following double integrals represents the volume of the solid bounded above by the graph of $z = 6 - x^2 - 2y^2$ and bounded below by the graph of $z = -2 + x^2 + 2y^2$?

(A) $4\int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{2}} (8 - 2x^2 - 4y^2)\ dy\ dx$

(B) $\int_{x=-2}^{x=2} \int_{y=-\sqrt{(4-x^2)/2}}^{y=\sqrt{(4-x^2)/2}} (8 - 2x^2 - 4y^2)\ dy\ dx$

(C) $4\int_{y=0}^{y=\sqrt{2}} \int_{x=-\sqrt{4-2y^2}}^{x=\sqrt{4-2y^2}} dx\ dy$

(D) $\int_{y=-\sqrt{2}}^{y=\sqrt{2}} \int_{x=-2}^{x=2} (8 - 2x^2 - 4y^2)\ dx\ dy$

(E) $2\int_{y=0}^{y=\sqrt{2}} \int_{x=0}^{x=\sqrt{4-2y^2}} (8 - 2x^2 - 4y^2)\ dx\ dy$

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21. Let a be a number in the interval $[0, 1]$ and let f be a function defined on $[0, 1]$ by

$$f(x) = \begin{cases} a^2 & \text{if } 0 \le x \le a, \\ ax & \text{otherwise.} \end{cases}$$

Then the value of a for which $\int_0^1 f(x)\,dx = 1$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

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22. If b and c are elements in a group G, and if $b^5 = c^3 = e$, where e is the unit element of G, then the inverse of $b^2cb^4c^2$ must be

(A) b^3c^2bc (B) $b^4c^2b^2c$ (C) $c^2b^4cb^2$ (D) $cb^2c^2b^4$ (E) cbc^2b^3

---

23. Let f be a real-valued function continuous on the closed interval $[0, 1]$ and differentiable on the open interval $(0, 1)$ with $f(0) = 1$ and $f(1) = 0$. Which of the following must be true?

I. There exists $x \in (0, 1)$ such that $f(x) = x$.
II. There exists $x \in (0, 1)$ such that $f'(x) = -1$.
III. $f(x) > 0$ for all $x \in [0, 1)$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

24. If A and B are events in a probability space such that $0 < P(A) = P(B) = P(A \cap B) < 1$, which of the following CANNOT be true?

(A) A and B are independent.
(B) A is a proper subset of B.
(C) $A \neq B$
(D) $A \cap B = A \cup B$
(E) $P(A)P(B) < P(A \cap B)$

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25. Let f be a real-valued function with domain $[0, 1]$. If there is some $K > 0$ such that $f(x) - f(y) \leq K|x - y|$ for all x and y in $[0, 1]$, which of the following must be true?

(A) f is discontinuous at each point of $(0, 1)$.
(B) f is not continuous on $(0, 1)$, but is discontinuous at only countably many points of $(0, 1)$.
(C) f is continuous on $(0, 1)$, but is differentiable at only countably many points of $(0, 1)$.
(D) f is continuous on $(0, 1)$, but may not be differentiable on $(0, 1)$.
(E) f is differentiable on $(0, 1)$.

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26. Let $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal if $\mathbf{v}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{v}_2 =$

(A) $\mathbf{i} + \mathbf{j} - \mathbf{k}$ (B) $\mathbf{i} - \mathbf{j} + \mathbf{k}$ (C) $\mathbf{i} + \mathbf{k}$ (D) $\mathbf{j} - \mathbf{k}$ (E) $\mathbf{i} + \mathbf{j}$

---

27. If the curve in the yz-plane with equation $z = f(y)$ is rotated around the y-axis, an equation of the resulting surface of revolution is

(A) $x^2 + z^2 = [f(y)]^2$
(B) $x^2 + z^2 = f(y)$
(C) $x^2 + z^2 = |f(y)|$
(D) $y^2 + z^2 = |f(y)|$
(E) $y^2 + z^2 = [f(x)]^2$

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28. Let A and B be subspaces of a vector space V. Which of the following must be subspaces of V?

I. $A + B = \{a + b: \ a \in A \text{ and } b \in B\}$
II. $A \cup B$
III. $A \cap B$
IV. $\{x \in V: \ x \notin A\}$

(A) I and II only
(B) I and III only
(C) III and IV only
(D) I, II, and III only
(E) I, II, III, and IV

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29. $$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{2^k} \right) =$$

(A) 0 (B) 1 (C) 2 (D) 4 (E) $+\infty$

---

30. If $f(x_1, \ldots, x_n) = \sum_{1 \le i < j \le n} x_i x_j$, then $\dfrac{\partial f}{\partial x_n} =$

(A) $n!$ (B) $\sum_{1 \le i < j < n} x_i x_j$ (C) $\sum_{1 \le i < j < n} (x_i + x_j)$ (D) $\sum_{j=1}^{n} x_j$ (E) $\sum_{j=1}^{n-1} x_j$

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31. If $f(x) = \begin{cases} \sqrt{1 - x^2} & \text{for } 0 \le x \le 1 \\ x - 1 & \text{for } 1 < x \le 2, \end{cases}$

then $\int_0^2 f(x)\,dx$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{1}{2} + \frac{\pi}{4}$

(D) $\frac{1}{2} + \frac{\pi}{2}$

(E) undefined

---

32. Let R denote the field of real numbers, Q the field of rational numbers, and Z the ring of integers. Which of the following subsets F_i of R, $1 \le i \le 4$, are subfields of R?

$$F_1 = \{a/b:\ a, b \in Z \text{ and } b \text{ is odd}\}$$
$$F_2 = \{a + b\sqrt{2}:\ a, b \in Z\}$$
$$F_3 = \{a + b\sqrt{2}:\ a, b \in Q\}$$
$$F_4 = \{a + b\sqrt[4]{2}:\ a, b \in Q\}$$

(A) No F_i is a subfield of R.

(B) F_3 only

(C) F_2 and F_3 only

(D) F_1, F_2, and F_3 only

(E) F_1, F_2, F_3, and F_4

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33. If n apples, no two of the same weight, are lined up at random on a table, what is the probability that they are lined up in order of increasing weight from left to right?

(A) $\frac{1}{2}$ (B) $\frac{1}{n}$ (C) $\frac{1}{n!}$ (D) $\frac{1}{2^n}$ (E) $\left(\frac{1}{n}\right)^n$

34. $\frac{d}{dx}\int_0^{x^2} e^{-t^2}\,dt =$

(A) e^{-x^2} (B) $2e^{-x^2}$ (C) $2e^{-x^4}$ (D) $x^2e^{-x^2}$ (E) $2xe^{-x^4}$

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35. Let f be a real-valued function defined on the set of integers and satisfying $f(x) = \frac{1}{2}f(x-1) + \frac{1}{2}f(x+1)$. Which of the following must be true?

I. The graph of f is a subset of a line.
II. f is strictly increasing.
III. f is a constant function.

(A) None
(B) I only
(C) II only
(D) I and II
(E) I and III

---

36. If F is a function such that, for all positive integers x and y, $F(x, 1) = x + 1$, $F(1, y) = 2y$, and $F(x+1, y+1) = F(F(x, y+1), y)$, then $F(2, 2) =$

(A) 8 (B) 7 (C) 6 (D) 5 (E) 4

---

37. If $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} = 9$, then $\det\begin{pmatrix} 3a & 3b & 3c \\ g-4a & h-4b & k-4c \\ d & e & f \end{pmatrix} =$

(A) -108 (B) -27 (C) 3 (D) 12 (E) 27

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38. $\lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n}\left[\left(\frac{3i}{n}\right)^2 - \left(\frac{3i}{n}\right)\right] =$

(A) $-\frac{1}{6}$ (B) 0 (C) 3 (D) $\frac{9}{2}$ (E) $\frac{31}{6}$

---

39. For a real number x, $\log(1 + \sin 2\pi x)$ is not a real number if and only if x is

(A) an integer

(B) nonpositive

(C) equal to $\frac{2n-1}{2}$ for some integer n

(D) equal to $\frac{4n-1}{4}$ for some integer n

(E) any real number

---

40. If x, y, and z are selected independently and at random from the interval [0, 1], then the probability that $x \geq yz$ is

(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $\frac{1}{4}$

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41. If $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, then the set of all vectors X for which $AX = X$ is

(A) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| \, a = 0 \text{ and } b \text{ is arbitrary} \right\}$

(B) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| \, a \text{ is arbitrary and } b = 0 \right\}$

(C) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| \, a = -b \text{ and } b \text{ is arbitrary} \right\}$

(D) $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

(E) the empty set

---

42. What is the greatest value of b for which any real-valued function f that satisfies the following properties must also satisfy $f(1) < 5$?

(i) f is infinitely differentiable on the real numbers;
(ii) $f(0) = 1$, $f'(0) = 1$, and $f''(0) = 2$; and
(iii) $|f'''(x)| < b$ for all x in $[0, 1]$.

(A) 1 (B) 2 (C) 6 (D) 12 (E) 24

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43. Let n be an integer greater than 1. Which of the following conditions guarantee that the equation

$x^n = \sum_{i=0}^{n-1} a_i x^i$ has at least one root in the interval (0, 1) ?

I. $a_0 > 0$ and $\sum_{i=0}^{n-1} a_i < 1$

II. $a_0 > 0$ and $\sum_{i=0}^{n-1} a_i > 1$

III. $a_0 < 0$ and $\sum_{i=0}^{n-1} a_i > 1$

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III

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44. If x is a real number and P is a polynomial function, then $\lim_{h \to 0} \dfrac{P(x+3h) + P(x-3h) - 2P(x)}{h^2} =$

(A) 0 (B) $6P'(x)$ (C) $3P''(x)$ (D) $9P''(x)$ (E) ∞

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45. Consider the system of equations

$$ax^2 + by^3 = c$$
$$dx^2 + ey^3 = f$$

where a, b, c, d, e, and f are real constants and $ae \neq bd$. The maximum possible number of real solutions (x, y) of the system is

(A) none (B) one (C) two (D) three (E) five

46. If $x^3 - x + 1 = a_0 + a_1(x - 2) + a_2(x - 2)^2 + a_3(x - 2)^3$ for all real numbers x, then (a_0, a_1, a_2, a_3) is

(A) $\left(1, \frac{1}{2}, 0, -\frac{1}{8}\right)$

(B) $(1, -1, 0, 1)$

(C) $(7, 6, 10, 1)$

(D) $(7, 11, 12, 6)$

(E) $(7, 11, 6, 1)$

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47. Let C be the ellipse with center $(0, 0)$, major axis of length $2a$, and minor axis of length $2b$. The value of $\oint_C x\,dy - y\,dx$ is

(A) $\pi\sqrt{a^2 + b^2}$

(B) $2\pi\sqrt{a^2 + b^2}$

(C) $2\pi ab$

(D) πab

(E) $\frac{\pi ab}{2}$

---

48. Let G_n denote the cyclic group of order n. Which of the following direct products is NOT cyclic?

(A) $G_{17} \times G_{11}$
(B) $G_{17} \times G_{11} \times G_5$
(C) $G_{17} \times G_{33}$
(D) $G_{22} \times G_{33}$
(E) $G_{49} \times G_{121}$

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49. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{4}(1 - x^2) & \text{if } -1 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the standard deviation of X ?

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{\sqrt{30}}{15}$ (D) $\frac{1}{\sqrt{5}}$ (E) 1

---

50. The set of all points (x, y, z) in Euclidean 3-space such that

$$\begin{vmatrix} 1 & x & y & z \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix} = 0$$

is

(A) a plane containing the points (1, 0, 0), (0, 1, 0), and (0, 0, 1)
(B) a sphere with center at the origin and radius 1
(C) a surface containing the point (1, 1, 1)
(D) a vector space with basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)}
(E) none of the above

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51. An automorphism ϕ of a field F is a one-to-one mapping of F onto itself such that $\phi(a + b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$ for all $a, b \in F$. If F is the field of rational numbers, then the number of distinct automorphisms of F is

(A) 0 (B) 1 (C) 2 (D) 4 (E) infinite

52. Let T be the transformation of the xy-plane that reflects each vector through the x-axis and then doubles the vector's length.

If A is the 2×2 matrix such that $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = A\begin{bmatrix} x \\ y \end{bmatrix}$ for each vector $\begin{bmatrix} x \\ y \end{bmatrix}$, then $A =$

(A) $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} \frac{\sqrt{2}}{2} & 1 \\ 1 & \frac{-\sqrt{2}}{2} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

(E) $\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$

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53. Let $r > 0$ and let C be the circle $|z| = r$ in the complex plane. If P is a polynomial function, then $\int_C P(z)\,dz =$

(A) 0
(B) πr^2
(C) $2\pi i$
(D) $2\pi P(0)i$
(E) $P(r)$

---

54 If f and g are real-valued differentiable functions and if $f'(x) \geq g'(x)$ for all x in the closed interval $[0, 1]$, which of the following must be true?

(A) $f(0) \geq g(0)$

(B) $f(1) \geq g(1)$

(C) $f(1) - g(1) \geq f(0) - g(0)$

(D) $f - g$ has no maximum on $[0, 1]$

(E) $\frac{f}{g}$ is a nondecreasing function on $[0, 1]$.

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55. Let p and q be distinct primes. There is a proper subgroup J of the additive group of integers which contains exactly three elements of the set $\{p, p + q, pq, p^q, q^p\}$. Which three elements are in J?

(A) pq, p^q, q^p
(B) $p + q, pq, p^q$
(C) $p, p + q, pq$
(D) p, p^q, q^p
(E) p, pq, p^q

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56. For a subset S of a topological space X, let $\mathrm{cl}(S)$ denote the closure of S in X, and let $S' = \{x: x \in \mathrm{cl}(S - \{x\})\}$ denote the derived set of S. If A and B are subsets of X, which of the following statements are true?

I. $(A \cup B)' = A' \cup B'$
II. $(A \cap B)' = A' \cap B'$
III. If A' is empty, then A is closed in X.
IV If A is open in X, then A' is not empty.

(A) I and II only
(B) I and III only
(C) II and IV only
(D) I, II, and III only
(E) I, II, III, and IV

57. Consider the following procedure for determining whether a given name appears in an alphabetized list of n names.

Step 1. Choose the name at the middle of the list (if $n = 2k$, choose the kth name); if that is the given name, you are done; if the list is only one name long, you are done. If you are not done, go to Step 2.

Step 2. If the given name comes alphabetically before the name at the middle of the list, apply Step 1 to the first half of the list; otherwise, apply Step 1 to the second half of the list.

If n is very large, the maximum number of steps required by this procedure is close to

(A) n
(B) n^2
(C) $\log_2 n$
(D) $n \log_2 n$
(E) $n^2 \log_2 n$

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58. Which of the following is an eigenvalue of the matrix

$$\begin{pmatrix} 2 & 1-i \\ 1+i & -2 \end{pmatrix}$$

over the complex numbers?

(A) 0 (B) 1 (C) $\sqrt{6}$ (D) i (E) $1+i$

---

59. Two subgroups H and K of a group G have orders 12 and 30, respectively. Which of the following could NOT be the order of the subgroup of G generated by H and K?

(A) 30 (B) 60 (C) 120 (D) 360 (E) Countable infinity

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60. Let A and B be subsets of a set M and let $S_0 = \{A, B\}$. For $i \geq 0$, define S_{i+1} inductively to be the collection of subsets X of M that are of the form $C \cup D$, $C \cap D$, or $M - C$ (the complement of C in M), where $C, D \in S_i$. Let $S = \bigcup_{i=0}^{\infty} S_i$. What is the largest possible number of elements of S?

(A) 4
(B) 8
(C) 15
(D) 16
(E) S may be infinite.

GO ON TO THE NEXT PAGE.

61. A city has square city blocks formed by a grid of north-south and east-west streets. One automobile route from City Hall to the main firehouse is to go exactly 5 blocks east and 7 blocks north. How many different routes from City Hall to the main firehouse traverse exactly 12 city blocks?

(A) $5 \cdot 7$

(B) $\frac{7!}{5!}$

(C) $\frac{12!}{7!5!}$

(D) 2^{12}

(E) $7!5!$

---

62. Let R be the set of real numbers with the topology generated by the basis $\{[a, b): a < b, \text{ where } a, b \in R\}$. If X is the subset $[0, 1]$ of R, which of the following must be true?

I. X is compact.
II. X is Hausdorff.
III. X is connected.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

GO ON TO THE NEXT PAGE.

63. Let R be the circular region of the xy-plane with center at the origin and radius 2.
Then $\int_R\int e^{-(x^2+y^2)}dx\,dy =$

(A) 4π

(B) πe^{-4}

(C) $4\pi e^{-4}$

(D) $\pi(1-e^{-4})$

(E) $4\pi(e-e^{-4})$

---

64. Let V be the real vector space of real-valued functions defined on the real numbers and having derivatives of all orders. If D is the mapping from V into V that maps every function in V to its derivative, what are all the eigenvectors of D?

(A) All nonzero functions in V

(B) All nonzero constant functions in V

(C) All nonzero functions of the form $ke^{\lambda x}$, where k and λ are real numbers

(D) All nonzero functions of the form $\sum_{i=0}^{k} c_i x^i$, where $k > 0$ and the c_i's are real numbers

(E) There are no eigenvectors of D.

GO ON TO THE NEXT PAGE.

65. If f is a function defined by a complex power series expansion in $z - a$ which converges for $|z - a| < 1$ and diverges for $|z - a| > 1$, which of the following must be true?

(A) $f(z)$ is analytic in the open unit disk with center at a

(B) The power series for $f(z + a)$ converges for $|z + a| < 1$.

(C) $f'(a) = 0$

(D) $\int_C f(z)dz = 0$ for any circle C in the plane.

(E) $f(z)$ has a pole of order 1 at $z = a$.

---

66. Let n be any positive integer and $1 \leq x_1 < x_2 < \ldots < x_{n+1} \leq 2n$, where each x_i is an integer. Which of the following must be true?

I. There is an x_i that is the square of an integer.
II. There is an i such that $x_{i+1} = x_i + 1$.
III. There is an x_i that is prime.

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST.

DO NOT USE INK

Use only a pencil with soft, black lead (No. 2 or HB) to complete this answer sheet. Be sure to fill in completely the space that corresponds to your answer choice. Completely erase any errors or stray marks.

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SIDE 1

1. NAME Enter your last name, first name initial (given name), and middle initial if you have one. Omit spaces, apostrophes, Jr., II., etc.

Last Name only (Family Name or Surname) - First 15 Letters | First Name Initial | Middle Initial

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

2.

YOUR NAME: (Print) ______ Last Name (Family or Surname) First Name (Given) M.I.

MAILING ADDRESS: (Print) ______ P.O. Box or Street Address

City ______ State or Province

Country ______ Zip or Postal Code

CENTER: ______ City ______ State or Province

Country ______ Center Number ______ Room Number

SIGNATURE: ______

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●.

YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

1–38	A B C D E
39–76	A B C D E
77–114	A B C D E

Item responses continued on reverse side.

3. DATE OF BIRTH

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