Math 7H: Honors Seminar

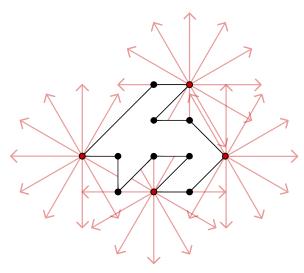
Homework 4: The Art Gallery Problem

Due Tuesday, week 5, at the start of class

Checkdown/also extra-credit problem.

So, this week is slightly weird. To get credit for this problem, just work on it; i.e. think about it, write down ideas, and otherwise spend a hour or two trying to figure out what's going on. Assignments that show effort (i.e. > 1/2 page of work and writing, coherent thoughts, good questions for me) will get credit. If you fully answer the problem, you get the extra-credit half-point as well!

1. (Fortress problem.) Take a polygon P with n sides. Consider the following task: we want to station observers at the vertices of P, such that they can guard the entire "outside" of P. In this situation, we assume that the guards cannot "see through" P's walls, and can only look out from their positions. For example, here is a polygon P being guarded by four guards:



A polygon with observers guarding its outside. Observers are denoted by red vertices; sample sight lines are drawn in pale red.

Basically, this is the art gallery problem, except we're guarding the outside instead of the inside!

Suppose that P is an arbitrary polygon with n vertices. What is the maximum number of guards needed to guard the exterior of P? Justify your answer; i.e. explain why you must be right!