Math 7H | Professor: Padraic Bartlett

# Homework 7: Electrical Networks and Random Walks

*Due Tuesday, week 8* | *UCSB*

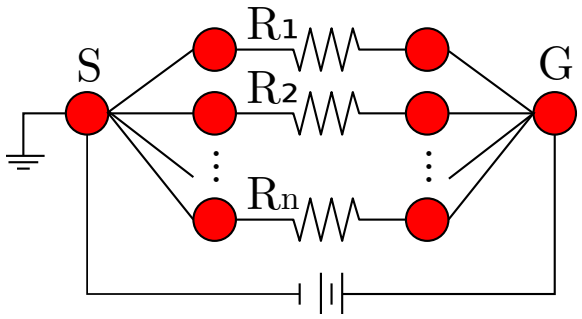
## 1 Checkdown Problems

1. Prove the claim we made in class: that in $\mathbb{Z}^2$, the number of edges connecting points at distance $n$ from the origin to points at distance $n+1$ is $8n+4$. (That is, the number of edges connecting the origin to the four points $(\pm 1, 0), (0, \pm 1)$ distance 1 from the origin was 4; the number of edges connecting those four points to $(\pm 2, 0), (0 \pm 2), (\pm 1, \pm 1)$ was 12; etc.)

## 2 Extra-Credit Problems

1. Prove the following claims about resistors we made in class:

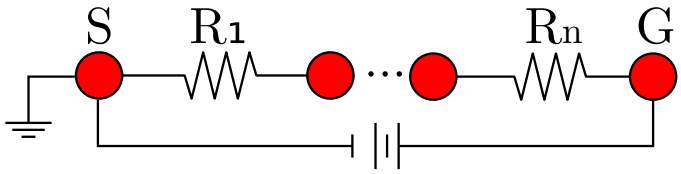
   (a) The effective resistance of the circuit below is the reciprocal of the sum of the reciprocals of the resistors in the circuit. In other words, the circuit




   has effective resistance given by the formula

$$\frac{1}{R_{\text{eff}}} = \sum_{i=1}^{n} \frac{1}{R_i}.$$

   (b) The effective resistance of the circuit below is the sum of the resistors in the circuit. In other words,




   has effective resistance given by the formula

$$R_{\text{eff}} = \sum_{i=1}^{n} R_i.$$

2. Prove Rayleigh's Monotonicity Theorem:

   **Theorem 1.** *If any of the individual resistances in a circuit increase, then the overall effective resistance of the circuit can only increase or stay constant; conversely, if any of the individual resistances in a circuit decrease, the overall effective resistance of the circuit can only decrease or stay constant.*

   *In specific, cutting wires (setting certain resistances to infinity) only increases the effective resistance, while fusing vertices together (setting certain resistances to 0) only decreases the effective resistance.*