

Homework 8: Algorithms

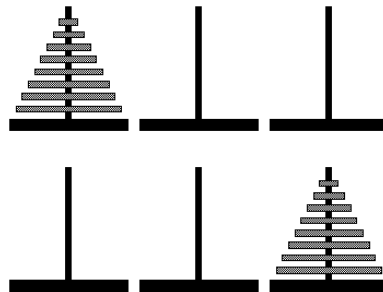
*Due Tuesday, week 9, at the start of class**UCSB***Checkdown / extra-credit problem**

This week is like some of the earlier weeks! To get credit for this problem, just work on it; i.e. think about it, write down ideas, and otherwise spend a hour or two trying to figure out what's going on. Assignments that show effort (i.e. $> 1/2$ page of work and writing, coherent thoughts, good questions for me) will get credit. If you fully answer the problem, you get the extra-credit half-point as well!

1. The Towers of Hanoi is the following puzzle: Start with 3 rods. On one rod, place n disks with radii $1, 2, \dots, n$, so that the disk with radius n is on the bottom, the disk with radius $n - 1$ is on top of that disk, and so on/so forth.

The goal of this puzzle is to move all of the disks from one rod to another rod, obeying the following rules:

- You can move only one disk at a time.
- Each move consists of taking the top disk off of some rod and placing it on another rod.
- You cannot place a disk A on top of any disk B with radius smaller than A .



- (a) How many disk-movements at most does it take you to solve this problem for $n = 1$? $n = 2$? $n = 3$? $n = 4$?
- (b) Write down an algorithm for solving this problem! (I.e. create step-by-step instructions to solve the Towers of Hanoi.)
- (c) What is the runtime of your algorithm? (Measure your runtime in terms of the total number of disks moved.)
- (d) A story, told by the mathematician Édouard Lucas, claimed that there was a monastery where monks were tirelessly solving a instance of the Tower of Hanoi problem made with 64 disks, and that when they finish their game the world will end. Suppose these monks can make an average of one move per second, and started around 1000BC. When will they complete their game (approximately?)