Math 7H	Professor: Padraic Bartlett

Homework 9: Latin Squares

Due Tuesday, week 10, at the start of class	*UCSB*

Checkdown problem

Prove one of the two problems below!

1. In class, we made the following claim. Prove it!

   Claim. Suppose that n is a prime number. Then $\langle \mathbb{Z}/n\mathbb{Z}, +, \cdot \rangle$ has the following property:

   For any $a, b \in \{0, \ldots n-1\}$, if $a \cdot b \equiv 0 \mod n$, then at least one of a, b are equal to 0.

2. In class, we made the following claim. Prove it!

   Claim. Suppose that n is a prime. Take any $a \in \mathbb{Z}/n\mathbb{Z}$. If $a \neq 0$. then there is some $b \in \mathbb{Z}/n\mathbb{Z}$ such that $a \cdot b = 1$.

Extra-credit problems

1. In class, every pair of orthogonal Latin squares were related to each other by simple row permutations: that is, if L and M were orthogonal, then we could turn L into M by switching the rows of L around.

   Prove that this is not true for all Latin squares! That is: find a pair of mutually orthogonal Latin squares L, M such that you cannot switch the rows of L around to get M.

2. Given a pair of mutually orthogonal Latin squares (A, B) of order m and another pair of mutually orthogonal Latin squares (C, D) of order n, create a pair of mutually orthogonal Latin squares (X, Y) of order mn.

3. Write a computer program that, given inputs n, k, will try to find k mutually orthogonal Latin squares of order n. For what values of n, k does your program finish running in (say) under half a hour?