Math 7H | Professor: Padraic Bartlett

Homework 2: Error-Correcting Codes

Due Tuesday, Week 2 | *UCSB 2015*

Try some of the problems below! As always, work on problems here until you've spent at least 90 minutes on this set. Show your work, so that I can see that you've spent time/effort on this!

1. Here's a fun problem from a UCSB mathematics Ph.D. student[1] from their dissertation:

   Question. You and two friends have been captured by eeeeevil logicians! They tell you ahead of time about the following puzzle they have for you:

   - You will all be led into a locked room.
   - Each person will have a hat placed on their head; hats are either black or white, and randomly decided for each person by flipping a fair coin.
   - No one can see their own hat.
   - Each person can see other people's hats.
   - You and your friends cannot communicate once in the room.
   - When the guards say so, you and your friends must all either guess (simultaneously) the color of their own hat, or say "pass."
   - If at least one person guesses correctly and no one is incorrect, you're free!
   - If anyone guesses incorrectly, you are sad/eaten by bears.

   Find a strategy that insures that on average, you are not eaten by bears three-quarters of the time.

2. Historically, one of the first codes developed was the Hamming $[7, 4]$ code. It works like this: take any string of four bits (i.e. any string of four 0's and 1's.) Turn this into a string of seven bits in the following way:

   - Place the bits of the original message, in order, in the slots $3, 5, 6, 7$.
   - In slot 1, put the parity[2] of the sum of the bits in slots $3, 5, 7$.
   - In slot 2, put the parity of the sum of the bits in slots $3, 6, 7$.
   - In slot 4, put the parity of the sum of the bits in slots $5, 6, 7$.

   For example, to encode the message 1010, we would first place

   $$\_\_1\_010;$$

[1] Todd Ebert, 1998. The silly framing is me.

[2] The parity of a number n is just $n \mod 2$. In other words, it is 1 if n is odd, and 0 if n is even.

then, because $1+0+0=1, 1+1+0=0, 0+1+0=1$, we would fill in the remaining slots to get

$$1011010.$$

This is a 2-ary code of length 7. Find its information density and its minimum distance.