Math 7H	Professor: Padraic Bartlett

Homework 8: Ramsey Theory

Due Tuesday, Week 9, at the start of class — *UCSB 2015*

In class, we mentioned that finding $R(3,3)$ can be thought of as a game:

- There are two players, Red and Blue. Their gameboard consists of some number n of points drawn on a plane. Players alternate turns, and Red starts first.
- On a given player's turn, they must connect two points that do not have a line drawn between them yet, with a line of their given color.
- The game ends when a monochromatic triangle is drawn on the board, in which case that player loses.

We proved in class that this game always ends with one player losing (i.e. no draws are possible) when $n \geq 6$ because $R(3,3) = 6$.

1. Suppose you play the game above on a gameboard with just **five** points. Prove that with optimal play from both players[1], this game will always end in a draw.
2. Assuming that both players play optimally, which player always wins: red or blue? (Warning: hard!)

[1] To illustrate this idea: tic-tac-toe is another example of such a game, where with both players playing optimally they can always force a draw.