Problem 1. \([\S 2.25.4(c)]\) Find the singularities of

\[ f(z) = \frac{z^2 + 1}{(z + 2)(z^2 + 2z + 2)} \]

and show that it is analytic except at those singularities.

Solution. We notice that since the product of analytic functions is analytic (at a point), and the reciprocal of an analytic function (that doesn’t vanish at the relevant point) is analytic, \(f(z)\) is analytic except at the roots of the denominator. We rewrite

\[ z^2 + 2z + 2 = (z + 1)^2 + 1 \]

and thus notice that the roots of the denominator are \([-2, -1 + i, -1 - i]\) and \(f\) is analytic everywhere except at these three points.