Problem 1. [§3.31.6] Given that the branch \( \log z = \ln r + i\theta \) \((r > 0, \alpha < \theta < \alpha + 2\pi)\) of the logarithmic function is analytic at each point \( z \) in the stated domain, obtain its derivative by differentiating each side of the identity
\[
e^{\log z} = z \quad (z \neq 0)
\]
and using the chain rule.

Solution. By the chain rule,
\[
\frac{d}{dz} e^{\log z} = \frac{d}{dz} \log z e^{\log z}.
\]
Thus
\[
\frac{d}{dz} \log z e^{\log z} = \frac{d}{dz} z = 1
\]
and hence
\[
\frac{d}{dz} \log z = \frac{1}{e^{\log z}} = \frac{1}{z}.
\]