Math 243C – Assignment 3  
Due 05/20/10

(1) Analyze the bifurcation that occurs when \( \mu = 0 \) for the following system:
\[
x_1' = x_2, \quad x_2' = -x_2 - \mu + x_1^2.
\]

(2) (a) Given that \( A = \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix} \), calculate \( L_A h(x) \), where \( h(x) = x^m e_j \), \( |m| = 2 \), and \( j = 1 \) or 2.
(b) Find a change of coordinates of the form \( x = y + h(y) \) which transforms the system
\[
x_1' = 3x_2 - x_1^2 + 7x_1x_2 + 3x_2^2, \\
x_2' = 2x_1 + 4x_1x_2 + x_2^2
\]
into the form
\[
y_1' = 3y_2 + O(|y|^3) \\
y_2' = 2y_1 + O(|y|^3).
\]

(3) Prove that both the Rayleigh equation
\[
\ddot{x} + \dot{x}^3 - \mu \dot{x} + x = 0
\]
and the Van der Pol equation
\[
\ddot{x} + \dot{x}(\mu - x^2) + x = 0,
\]
undergo Hopf bifurcations at \( \mu = 0 \). In both cases, sketch the phase portraits for \( \mu < 0 \), \( \mu = 0 \), and \( \mu > 0 \). State what type of Hopf bifurcation takes place.