Name: Solutions

Perm

Final
Math 34A

Summer 2007
Session B
Instructor: Tom Howard
Time: 11:00 AM - 12:10 PM

Please TURN OFF all cell phones

NO textbooks, notebooks, or calculators allowed. ONE index card is allowed.

You may leave logs, fractions, square roots, π, e, etc. in your final answer. You may not leave unperformed derivatives.

This exam is graded out of 100 points, with 15 bonus points available. Problem 7 is a bonus problem.

Please be neat. Be sure your answers are carefully boxed. If you need to write on the back of a page please indicate which problem you are working on.

You must show work to get credit. The more work you show, the more partial credit I can give you. “X” out any work you don’t want considered. There is no partial credit on the bonus problems.

Check your answers. Are your answers reasonable? Are you answering the question? Check your units. Are your variables properly labeled?
1) [5 points] a) Find the equation of the line passing through the points \((1, 1)\) and \((-2, 7)\).

\[
m = \frac{7 - 1}{-2 - 1} = \frac{6}{-3} = -2
\]

\[
y - 1 = -2(x - 1)
\]

\[
y - 1 = -2x + 2
\]

\[
y = -2x + 3
\]

[5 points] b) Find the equation of the line with slope \(\frac{1}{3}\) passing through the origin.

\[
y = \frac{1}{3}x
\]
2) [5 points] a) Let $f(x) = (x + b)^2$. Find $f'(x)$. [b is constant]

$$f(x) = (x + b)^2 = x^2 + 2bx + b^2$$

$$f'(x) = 2x + 2b$$

[5 points] b) Let $f(x) = \frac{8}{x^2} + \sqrt{x}$. Find $f'(4)$.

$$f'(x) = \frac{8}{x^2} + \sqrt{x} = 8x^{-2} + x \cdot \frac{x}{2}$$

$$f'(4) = -16 \cdot 4^{-3} + \frac{1}{2} \cdot 4^{-\frac{1}{2}}$$

$$f'(4) = -\frac{1}{4^3} + \frac{1}{2} \cdot \frac{1}{4} = 0$$

[5 points] c) Let $f(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$. Find $f'(x)$.

$$f(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$$

$$f'(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$$
The function \( f(x) = \frac{e^x}{1+e^x} \) is frequently used for modelling population growth. Given that \( f'(0) = \frac{1}{4} \), find the equation of the tangent line at \( x = 0 \).

Tangent Line Approximation:

\[
y = f(c) + f'(c)(x - 0)
\]

\[f'(c) = \frac{1}{4} \text{ given.}\]

\[f(c) = \frac{e^0}{1+e^0} = \frac{1}{1+1} = \frac{1}{2}\]

\[
y = \frac{1}{2} + \frac{1}{4}x
\]
4) Suppose you have a tank of water. The height of the water after \( t \) days is 
\[ h(t) = -t^3 + 3t^2 + 10t \] meters.

[10 points] a) What is the average rate of change of the height between \( t = 1 \) and \( t = 5 \)?

\[
\frac{\Delta h}{\Delta t} = \frac{h(5) - h(1)}{5-1} = \frac{\left[-5^3 + 3(5)^2 + 10(5)\right] - \left[-1^3 + 3(1)^2 + 10(1)\right]}{4} \\
= \frac{\left[-125 + 75 + 50\right] - \left[-1 + 3 + 10\right]}{4} = \frac{0 - 12}{4} = -3 \text{ meters/day}
\]

[10 points] b) How quickly is the water level rising at \( t = 1 \)?

\[ h'(t) = -3t^2 + 6t + 10 \]

\[ h'(1) = -3 + 6 + 10 = 13 \text{ meters/day} \]

[5 points] c) When is the water level rising most quickly?

\[ h''(t) = -6t + 6 = 0 \]

\[ t = 1 \text{ day} \]
[20 points] 5) You’re in charge of planning a concert for a popular band. You estimate that if you charge $150 you’ll sell 250 tickets, and that for every $2 you raise the price, 10 fewer people will attend. What price should you charge per ticket to maximize your revenue?

\[ p = \text{revenue} \]
\[ c = \text{cost} \]
\[ n = \text{number sold} \]
\[ p = n \cdot c \]
\[ n = 250 - 10 \left( \frac{c - 150}{2} \right) \]
\[ n = 250 - 5c + 750 \]
\[ n = 1000 - 5c \]
\[ p = (1000 - 5c) \cdot c \]
\[ p = 1000c - 5c^2 \]

\[ p'(c) = 1000 - 10c = 0 \]

\[ c = 100 \]

\[ \boxed{100} \]

$100$
6) Consider the following graph:

\[ y = f(x) \]

[4 points] a) For which values of \( x \) is the derivative positive?
\[ 0 < x < 2 \]

[4 points] b) For which values of \( x \) is the derivative zero?
\[ x = 0 \quad x = 2 \]

[4 points] c) For which values of \( x \) is the second derivative negative?
\[ x > 1 \]

[4 points] d) For which values of \( x \) is the second derivative zero?
\[ x = 1 \]

[4 points] e) If you used the tangent line at \( x = 1 \) to approximate \( f(1.2) \), would you overestimate it or underestimate it?
\textit{overestimate}
[Bonus] 7) For the following problems, you’ll need to rewrite the function before you can differentiate it.

[Bonus 5 points] a) Let \( f(x) = e^{x+2} \). Find \( f'(x) \).

\[
\frac{d}{dx} (e^{x+2}) = \frac{d}{dx} (e^x e^2) = e^2 \frac{d}{dx} (e^x) = e^2 e^x = e^{x+2}
\]

[Bonus 5 points] b) Find \( \frac{d}{dx} (\sqrt{4x}) \).

\[
\frac{d}{dx} (\sqrt{4x}) = \frac{d}{dx} (\sqrt{4} \sqrt{x}) = \frac{d}{dx} (2 \sqrt{x}) = 2 \frac{d}{dx} (x^{1/2}) = 2 \left( \frac{1}{2} x^{-\frac{1}{2}} \right) = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}
\]

[Bonus 5 points] c) Let \( f(x) = \frac{x^2 - 1}{x+1} \). Find \( f'(2) \).

\[
f'(x) = \frac{x^2 - 1}{x+1} = \frac{(x-1)(x+1)}{x+1} = x-1
\]

\[
f'(2) = 1, \quad f''(2) = 1
\]