Name __________________________

Perm _____________________________

Midterm
Math 34A

Summer 2007
Session B
Instructor: Tom Howard
Time: 11:00 AM - 12:10 PM

Please TURN OFF all cell phones

NO textbooks, notebooks, or calculators allowed. ONE index card is allowed.

You may use the table of logs on the next page. It is only required for problem 4. For other problems, you may leave logs, fractions, square roots, \( \pi \), etc. in your final answer.

This exam is graded out of 100 points, with 15 bonus points available. Problems 6b and 7 are bonus problems.

Please be neat. Be sure your answers are carefully boxed. If you need to write on the back of a page please indicate which problem you are working on.

You must show work to get credit. The more work you show, the more partial credit I can give you. “X” out any work you don’t want considered. There is no partial credit on the bonus problems.

Check your answers. Are you answering the question? Check your units. Are your variables properly labeled?
[15 points] 1) Solve for $x$ and $y$

\[
\begin{align*}
5x + 4y &= 2 \\
2x + y &= -1
\end{align*}
\]

(1) $5x + 4y = 2$

(2) $2x + y = -1$

Solve for $y$ in (2)

$y = -1 - 2x$

Plug into (1)

$5x + 4(-1 - 2x) = 2$

$5x - 4 - 8x = 2$

$-3x = 6$

$x = -2$

But,

$y = -1 - 2(-2) = -1 + 4 = 3$

\[
\begin{array}{c}
x = -2, \\
y = 3
\end{array}
\]
[15 points] 2) 100 sheets of paper costs $5 before tax. Sales tax is 20%. Every Monday, Pete's Paper Emporium has a 40% off sale. How much money will it cost to buy 500 sheets of paper on Monday, after tax?

\[ \text{price of 100 sheets} \times \frac{3}{5} \times \frac{12}{10} \times \frac{5}{5} \times 5 \]

\[ = 5 \times \frac{3}{5} \times \frac{6}{5} \times 5 \]

\[ = 3.6 = 18 \]

\[ \boxed{$18} \]
3) A cylindrical grain silo has a height of 30 meters. The grain is harvested from the fields and the silo is filled. The grain is used at a rate of 10 cubic meters per day.

[15 points] a) Express the area of the floor of the silo in terms of the number of days the grain will last.

\[ V = \text{volume} \ (m^3) \]
\[ A = \text{area} \ (m^2) \]
\[ h = \text{height} = 30 \ m \]
\[ T = \text{number of days lasted.} \]
\[ V = A \cdot h = 30A \]

\[ (\text{volume used}) = (\text{volume used each day}) \cdot (\text{days of use}) \]
\[ V = 10 \cdot T \]
\[ 30A = 10T \]
\[ A = \frac{1}{3} T \]

[5 points] b) How large should the floor be if the silo’s grains must last 4 months? [Assume 1 month = 30 days]

\[ T = 4 \cdot 30 = 120 \]
\[ A = \frac{1}{3} \cdot 120 = 40 \]
\[ 40 \ m^2 \]
4) Use the log table to calculate the following. Briefly explain your steps.

[5 points] a) \( \log(27.5) \)

\[
\log(27.5) = \log(2.75 \times 10^1) \\
= 1 + \log(2.75) \\
= 1 + 0.4393 \\
= \boxed{1.4393}
\]

[5 points] b) \( 10^{2.617} \)

\[
10^{2.617} = 10^{2 + 0.6170} \\
= 10^2 \times 10^{0.6170} \\
= 10^2 \times \text{antilog}(0.6170) \\
= 100 \times 4.14 = \boxed{414}
\]

[5 points] c) \( \sqrt{1.77} \)

\[
A = \sqrt{1.77} = (1.77)^{\frac{1}{2}} \\
\log(A) = \log((1.77)^{\frac{1}{2}}) \\
= \frac{1}{2} \log(1.77) \\
= \frac{1}{2} \times 0.2480 = 0.1240 \\
A = \text{antilog}(0.1240) = \boxed{1.33}
\]
[15 points] 5) Solve for $x$. Leave logs in your answer.

$5^x = 3^{-x+2}$

Take log of both sides

$\log (5^x) = \log (3^{-x+2})$

$x \log (5) = (-x+2) \log (3)$

$x \log (5) = -x \log (3) + 2 \log (3)$

$x [\log (5) + \log (3)] = 2 \log (3)$

$x = \frac{2 \log (3)}{\log (5) + \log (3)}$
6) A space explorer has discovered a new planet and wants to determine how long one day lasts on this planet (not necessarily the same as an Earth day). He has a sample of a radioactive substance with a half-life of 4 hours. He waits until dawn and observes that he has 16 grams. He waits through day, and at the following dawn he has 2 grams left.

[20 points] a) How long is one day on this planet?

\[ A = A_0 \left( \frac{1}{2} \right)^{t/4} \]

\[ 2 = 16 \left( \frac{1}{2} \right) \]

\[ \frac{1}{8} = \left( \frac{1}{2} \right)^{t/4} \]

\[ \log \left( \frac{1}{8} \right) = \log \left( \frac{1}{2} \right) \cdot \frac{t}{4} \]

\[ t = \frac{4 \log \left( \frac{1}{8} \right)}{\log \left( \frac{1}{2} \right)} = \frac{4 \log (8)}{\log (2)} = \frac{4 \log (2^3)}{\log (2)} = \frac{4 \cdot 3 \log (2)}{\log (2)} = 12 \text{ hours} \]

[OR, halving 16 three times gives 2. So 3 half-lives passed. \[ 3.4 = 12 \text{ hours} \]

[Bonus 5 points] b) He checks again at dusk, and now there’s 1 gram left. Is it winter or summer on this planet? Explain your reason.

half the sampled decay, so one half-life passed. That’s 4 hours of daylight, so night lasts 8 hours. If nighttime is longer than daytime, it’s \text{winter}.
[Bonus] 7) Sarah is playing a game of darts. She gets 25 points for hitting the bullseye and 10 points for hitting elsewhere on the target. She hits with 10 darts and scores 160 points.

[Bonus 5 points] a) How many bullseyes did she get?

\[ x = \text{number of bullseyes}, \quad y = \text{number of other hits}. \]

\[ (1) \quad 25x + 10y = 160 \]
\[ (2) \quad x + y = 10 \]

So \[ y = 10 - x \]

\[ 25x + 10(10-x) = 160 \]
\[ 15x = 60, \quad x = \boxed{4} \]

[Bonus 5 points] b) Jake claims that last week he scored 170 points with his 10 hits. Sarah suspects that he's lying. Is he? Explain your reason.

\[ x, y \text{ as above}. \]

\[ (1) \quad 25x + 10y = 170 \]
\[ (2) \quad x + y = 10 \]

So \[ y = 10 - x \]

\[ 25x + 10(10-x) = 170 \]
\[ 15x = 70 \]

\[ x \text{ is not a whole number.} \]

\[ \boxed{\text{Jake lied}} \]