Here are some short notes on Sections 5.1 - 5.5, 8.1 - 8.5 in your eBook. The best indication of what will be on the test is what you have seen in lecture and on the homework. No calculators will be allowed. You will be allowed one notecard. Please remember your TARDIS code. I suggest reading your syllabus for more information.

You should...

- Know how to find the area between two functions
- Know how to find the volume of a given region using cross-sectional areas (sometimes called “washer” method)
- Know how to find the volume of a given region using cylindrical method
- Know how to find work
- Know how to find the average value of a function
- Know how to find the length of a curve
- Know how to find the surface area of a given region
- Know applications to physics and engineering
- Know applications to biology and economics
- Know how to verify if a given function is a probability density function
- Know how to find the mean of a probability density function
- Know how to find probability of a given scenario

5.1 - Areas Between Curves

The area of the region bounded by two curves is given by

\[ A = \int_{a}^{b} (\text{top} - \text{bottom})\,dx \]

or

\[ A = \int_{c}^{d} (\text{right} - \text{left})\,dy. \]
5.2 - Volumes (Cross-Sectional Area or “Washer” method)

The volume of a solid with cross-sectional areas perpendicular to the $x$-axis and rotated about the $x$-axis is given by

$$V = \int_{a}^{b} A(x)dx.$$ 

Some helpful steps:

1. Identify what the cross-sectional area looks like (i.e. if it’s a circle, $A = \pi r^2$; if it’s a square, $A = s^2$; etc.).
2. Express your area from (1) in terms of the given values.
3. Integrate what you found in (2).

5.3 - Volumes (Cylindrical Method)

The volume of a cylinder is given by

$$V = 2\pi RH.$$ 

If we wish to find the volume of a solid obtained by rotating a function $y = f(x)$ about the $y$-axis, it is given by

$$V = 2\pi \int_{a}^{b} xf(x)dx.$$ 

In this particular case, $R = x$ and $H = f(x)$. If a solid is rotated about a vertical line other than the $y$-axis, $R$ will change.

5.4 - Work

Work is given by

$$W = \text{Force} \times \text{distance} = \text{mass} \times \text{acceleration} \times \text{distance}$$

Hooke’s Law is given by

$$f(x) = kx$$

where $k$ is the spring constant.

5.5 - Average Value of a Function

The average value of a function $f$ on the interval $[a, b]$ is given by

$$f_{\text{ave}} = \frac{1}{b-a} \int_{a}^{b} f(x)dx.$$
8.1 - Arc Length
If $a \leq x \leq b$, then the length of a given curve is given by
$$L = \int_{a}^{b} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx.$$ 
If $c \leq y \leq d$, then the length of a given curve is given by
$$L = \int_{c}^{d} \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy.$$ 

8.2 - Area of a Surface of Revolution
If we are rotating a curve about the $x$-axis, then the surface area is given by
$$S = \int 2\pi y \, ds.$$ 
If we are rotating a curve about the $y$-axis, then the surface area is given by
$$S = \int 2\pi x \, ds.$$ 
$ds$ is the arc length. We have two choices for $ds$: If $a \leq x \leq b$, then $ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx$; if $c \leq y \leq d$, then $ds = \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy$.

8.3 - Applications to Physics and Engineering
The *Hydrostatic Force* exerted by a fluid on a surface is given by
$$F = \rho g A d$$
and the *Hydrostatic Pressure* is given by
$$P = \frac{F}{A} = \rho g d.$$
If you are given masses on the $x$-axis, the *center of mass* is given by
$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}.$$ 
If you are given masses on the $(x, y)$ plane, the *center of mass* is given by $(\bar{x}, \bar{y})$ where
$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$
$$\bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}.$$
If you are given a region/plate and want to find the mass,

\[ m = \rho A = \rho \int_{a}^{b} f(x)dx. \]

If you are given a region/plate, the centroid is given by \((\bar{x}, \bar{y})\) where

\[ \bar{x} = \frac{1}{A} \int_{a}^{b} xf(x)dx \]
\[ \bar{y} = \frac{1}{A} \int_{a}^{b} \frac{1}{2} [f(x)]^2 dx \]

where \(A\) is the area of the given region. (Note: we did a quiz where we had two different functions, so \(\bar{x} = \frac{1}{A} \int_{a}^{b} x(f(x) - g(x))dx\) and \(\bar{y} = \frac{1}{A} \int_{a}^{b} \frac{1}{2} [(f(x))^2 - (g(x))^2]dx\).)

8.4 - Applications to Economics and Biology

The consumer surplus is given by

\[ \int_{0}^{X} [p(x) - P]dx \]

where \(p(x)\) is the demand function, \(X\) is the current commodity available, and \(P = p(X)\).

The producer surplus is given by

\[ \int_{0}^{X} [P - p_s(x)]dx \]

where \(p_s\) is the supply function, \(X\) is the current commodity available, and \(P = p_s(X)\).

8.5 - Probability

There are two requirements for a function \(f(x)\) be a probability density function:

1. \(f(x) \geq 0\) for all \(x\).
2. \(\int_{-\infty}^{\infty} f(x) = 1\).

(1) says our function must always be positive, and (2) says the probability over the entire domain adds up to 1.

If \(f(x)\) is a probability density function, the probability that \(X\) is between \(a\) and \(b\) is given by

\[ P(a \leq X \leq b) = \int_{a}^{b} f(x)dx. \]

The mean of a probability density function is given by

\[ \mu = \int_{-\infty}^{\infty} xf(x)dx. \]