Discussion time (circle one):
8am; 4pm; 5pm; 7pm
Total Score: 100
Your Scores: 1. .......  2. .......  3. .......  4. .......  5. .......  6. .......

Your Total ........................................

Books, notes are NOT allowed. Continue on the back of the page if more space is needed. READ the problems carefully. No calculators are allowed.

1. (40 pts) Find $f'(x)$ for each of the following
(a) $f(x) = (x - \cos 3x)^{3/2}$

$$f'(x) = \frac{3}{2} \left( x - \cos 3x \right)^{1/2} \left( 1 + 3 \sin (3x) \right)$$

(b) $f(x) = \ln(x^6 + 1)$

$$f'(x) = \frac{1}{x^6 + 1} \cdot 6x^5$$

(c) $f(x) = x^x$

$$\ln f = \ln x^x = x \ln x$$

$$f' \over f = \ln x + x \cdot {1 \over x}$$

$$f'(x) = x^x \left( 1 + \ln x \right)$$
d) \( f(x) = x \tan^{-1} \sqrt{x} \)

\[
f'(x) = \tan^{-1} \sqrt{x} + x \frac{1}{1 + x} \cdot \frac{1}{2} x^{-\frac{1}{2}}
\]

\[
= \tan^{-1} \sqrt{x} + \frac{\sqrt{x}}{2(1+x)}
\]

e) Find \( f^{(45)}(x) \) for \( f(x) = \sin 2x \).

\[
45 = 4 \times 11 + 1 \quad f'(x) = 2 \cos 2x
\]

\[
f^{(45)}(x) = 2^{45} \cos 2x
\]

2. (10 pts) Find the absolute maximum and minimum values of the function \( h(s) = s^3 - 12s + 3 \) on the interval \([-5, 3]\).

\[
h'(s) = 3s^2 - 12 = 3(s^2 - 4)
\]

\[
h'(s) = 0 \quad \Rightarrow \quad s = \pm 2
\]

Now \( h(-5) = -62 \quad h(-2) = 19 \)

\( h(2) = -13 \quad h(3) = -6 \)

So the absolute max value is 19

the absolute min value is -62.
3. (15 pts) A moving particle has its position given by the function

\[ s = 2t^3 - 6t, \]

where \( t \) is the time after the motion starts and measured in seconds, and \( s \) in feet.
a). When is the particle at rest?
b). When is the particle moving forward?
c). Find the total distance traveled during the first 2 seconds.

\[ s'(t) = 6t^2 - 6 = 6(t^2 - 1) = 6(t+1)(t-1) \]

a) The particle is at rest when \( t = 1 \)
b) The particle is moving forward when \( t > 1 \).
c) Total distance during the first 2 seconds is

\[ |s(1) - s(0)| + |s(2) - s(1)| \]

\[ = | -4 - 0 | + | 4 - (-4) | \]

\[ = 12 \]
4. (15 pts) Find the equation of the tangent line to the curve
\[ 2(x^3 + y^3) = 9xy \]
at \((1, 2)\).

By implicit differentiation,
\[
2(3x^2 + 3y^2y') = 9(y + xy')
\]

\[ 6y^2y' - 9xy' = 9y - 6x^2 \]

\[ y' = \frac{9y - 6x^2}{6y^2 - 9x} = \frac{3y - 2x^2}{2y^2 - 3x} \]

\[ y' \big|_{(1, 2)} = \frac{3 \cdot 2 - 2}{2 \cdot 2^2 - 3} = \frac{4}{5} \]

So the slope is \( \frac{4}{5} \).

The equation of the tangent line at \((1, 2)\) is
\[ y - 2 = \frac{4}{5} (x - 1) \]
5. (10 pts) Given \( f(x) = x^3 - x^2 - x + 1 \), fill in the following table.

- Increasing on \( X > 1 \), \( X < -\frac{1}{3} \)
- Decreasing on \( (-\frac{1}{3}, 1) \)
- Concave up on \( X > \frac{1}{3} \)
- Concave down on \( X < \frac{1}{3} \)
- Inflection point \( X = \frac{1}{3} \)

\[
f'(x) = 3x^2 - 2x - 1 = (3x + 1)(x - 1)
\]

\[
f''(x) = 6x - 2
\]

\[
\begin{array}{c|c|c|c|c|c}
& & & & & \\
& - & - & - & + & + \\
\hline
\frac{-1}{3} & 0 & - & + & + & 3x + 1 \\
\hline
- & + & + & + & x - 1 \\
\hline
\frac{-1}{3} & 0 & + & + & f'(x)
\end{array}
\]

6. (10 pts) A ladder of 5 feet long rests against a vertical wall, with the bottom of the ladder 3 feet away from the wall. The bottom of the ladder then began to slide away from the wall at an initial speed of 3 ft/s, how fast is the top of the ladder coming down at that moment?

Given \( \frac{dx}{dt} = 3 \) ft/s

Find \( \frac{dy}{dt} \) when \( X = 3 \) ft

Note that \( x^2 + y^2 = 5^2 \)

\[
2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0
\]

When \( x = 3 \), \( y = \sqrt{5^2 - 3^2} = 4 \) ft

So \( \frac{dy}{dt} = -\frac{3}{4} \) ft/s