Your Name: 
Your Perm Number: 
Your Signature: 

1. Be sure to write down all the steps.

2. Write cleanly and clearly!

3. The exam must be done individually. No team work is allowed. Do not consult anyone about the problems on the exam before you turn in your exam.

4. The exam is due at 4:00 pm on Friday, September 10. Please leave your exam in Prof. Ye’s mailbox in the mailroom of the math. department. (Please keep a backup copy of your exam.) You can also type up or scan your exam and email it to yer@math.ucsb.edu The deadline for emailing your exam is 8:00 pm on Friday, September 10.
You need to provide adequate reasoning for your computations in all the following problems.

1. (17 points) Determine all the isolated singularities of each of the following functions and compute the residue at each singularity.
   1) \( \frac{\cos z}{z(z-\frac{\pi}{2})(e^z-1)} \).
   2) \( \cot(z^2) \).

2. (17 points) Evaluate the following integral by means of the Cauchy residue theorem. The contour is positively oriented.

\[
\int_{|z|=4} \frac{z + 1}{z^4 - 1} \, dz
\]

3. (17 points) Find

\[
\int_0^{2\pi} \frac{\sin^2 \theta}{2 + \cos \theta} \, d\theta.
\]

4. (17 points) Find

\[
p.v. \int_{-\infty}^{\infty} \frac{x^2 e^{-ix}}{(x^2 + 1)(x - 1)} \, dx
\]

5. (17 points) Find

\[
p.v. \int_0^{\infty} \frac{x^{\alpha - 1}}{x^4 - 1} \, dx,
\]

where \( 0 < \alpha < 1 \).