

Final Exam
Math 122B
September 2011
Prof. Rick Ye

Your Name:
Your Perm Number:
Your Signature:

- 1.
- 2.
- 3.
- 4.

Total Score:

1. Be sure to write down all the steps.
2. Write cleanly and clearly!
3. The exam must be done individually. No team work is allowed. Do not consult anyone about the problems on the exam before you turn in your exam.
4. The exam is due at 6:00 pm on Thursday, September 9. Please slide your exam underneath the door into Prof. Ye's office SH6509 or leave it in Prof. Ye's mailbox in the mailroom of the math. department. (Please keep a backup copy of your exam.) You can also type up or scan your exam and email it to *yer@math.ucsb.edu*

You need to provide adequate reasoning for your computations in all the following problems.

1. (25 points) Determine the types of all the isolated singular points of each of the following functions and compute the residue at each singular point.

1)

$$\frac{\cos z}{z(z - \pi/2)(e^z - 1)}. \quad (0.1)$$

2)

$$\frac{z^2 + 2z + 1}{(z + 1)^3(z - 1)^2 \sin z}. \quad (0.2)$$

2. (25 points) Evaluate the following integral by means of the Cauchy residue theorem. The contour is positively oriented.

$$\int_{|z|=4} \frac{z^2 + 1}{(z^3 + 8)^2} dz \quad (0.3)$$

3. (25 points) Find

$$\int_0^{2\pi} \frac{\sin^2 \theta}{2 - \cos \theta} d\theta. \quad (0.4)$$

4. (25 points) Find

$$p.v. \int_{-\infty}^{\infty} \frac{x^2 e^{ix}}{(x^2 + 1)(x + 2)} dx \quad (0.5)$$