MATH 241C Topics in Differential Geometry
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Topics in Mathematical Theory of General Relativity

A number of fundamental results have been obtained in the mathematical theory of general relativity and related geometric fields. These results play an important role in the theory itself, and have also triggered new developments in geometric analysis. In this course we are planning to cover the following topics (not necessarily in this order):
1. First variation and second variation of area.
2. Toplogy and geometry of stable minimal surfaces in 3-manifolds of nonnegative scalar curvature. This is an independent direction, but is intimately related to the topics of asymptotically flat manifolds.
3. ADM mass and positive mass theorems of Schoen-Yau and Witten. The former uses minimal surfaces and nonlinear analysis, while the latter is based on spin geometry and linear analysis.
4. Constructions of unique foliation by constant mean curvature spheres on asymptotically flat manifolds, which provides a unique geometric structure and defines center of gravity. Here two fundamental results were obtained by Ye and Huisken-Yau independently. Huisken-Yau used the mean curvature flow, while Ye used an elliptic perturbation method. We'll present Ye's construction.
5. Topology, geometry of stable surfaces of constant mean curvature and Hawking mass. Hawking mass is an important concept of quasi-local mass which serves to localize ADM mass.

We'll cover as many topics on the above list as time allows. The above list also serves as a guideline for further reading.

References
1. "The Structure of Complete Stable Minimal Surfaces in 3-Manifolds of

Non-Negative Scalar Curvature", D. Fischer-Colbrie and R. Schoen, Communication in Pure and Appl. Math. **33**(1980), 199-211.
2. "The Yamabe problem", J. Lee and T. Parker, Bull. Amer. Math. Soc. **17**(1987), 37-91.
More references will be provided later.