Math 117: Homework 3

Instructor: Xin Zhou

Due time: Oct 19th, in class

All homework will come from the textbook (2013 Edition). The total grade will depend on a selection of the assigned problems to be graded and whether you finish all assignment.

1. 5.6, 7.4(b)(explain your answer), 8.1(a)(d), 8.2(b)(e), 8.4, 8.7(b) (Note that in Chap 8, all the proofs should be only based on the definition).

2. Show that if $S \subset \mathbb{Z}$ is bounded above then $S$ has a maximum, i.e., $\sup S \in S$.

*3 (This one is not required to submit, but you should think about it.) Show that the Archimedean Property of the real numbers holds if and only if the set of natural numbers $\mathbb{N}$ is unbounded above.

*4 (This one is not required to submit, but you should think about it.) Let $x \in \mathbb{R}$. We define the Greatest Integer function of $x$ as the largest integer $m \in \mathbb{Z}$ such that $m \leq x$; it is denoted by $[x]$ and satisfies the inequality

$$[x] \leq x < [x] + 1.$$ 

For instance, $[0.1] = 0$, $[1.1] = 1$, $[-2.1] = -3$, $[5] = 5$, $[-4] = -4$, etc. Show that for any real number $x \in \mathbb{R}$, $[x]$ exists. **Hint:** Consider the set

$$S = \{n \in \mathbb{Z}: n \leq x\}.$$ 

Then consider two cases depending on whether $x \geq 0$ or $x < 0$. In the case when $x < 0$, use the Archimedean property of $\mathbb{R}$ to show that $S$ is nonempty. Then apply the completeness axiom and finally show that $\sup S = [x]$, i.e., show that $\sup S$ is an integer (this follows from Problem 4) and it satisfies

$$\sup S \leq x < \sup S + 1.$$