THE DIAMETER OF THE ASSOCIAHEDRA

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Abstract. (add)

1. Tree Distances

In Nebraska I chatted with Patrick Dehornoy about calculating the diameter of the associahedra. The upper bound is known to be $2n-6$ for large values of n and during the week, I was able to give a geometric interpretation of the Sleater-Tarjan-Thurston proof. Basically, the worst case scenario is where the curvature is spread out. Because curvature in this context occurs in quanta, when there are many vertices, there must be (many) vertices of curvature 0. The short curves then pull all the curvature towards these vertices.

2. Random notes

There's a weird resonance between the fanned from a vertex triangulations that are the cone points of many of the shortest paths in the associahedra and the boundary parallel apartments that are the ones Tom and I keyed in on to show that there are no loops of length less than π in his complex for the braid groups. Is there a way to match these up? The apartments that are kept are the ones that correspond to plane trees and plane trees correspond to triangulations. Is there a change of n in the process? Can both maps be made precise? Can the CAT(0) question be reformulated into a question about associahedra?

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n	-2	-1	0	1	2	3	4	5	6	7	8	9	10
$2n$	-4	-2	0	2	4	6	8	10	12	14	16	18	20
$2n + \lfloor \frac{12}{n+5} \rfloor$	0	1	2	4	5	7	9	11	13	15	16	18	20
actual diameter	0	1	2	4	5	7	9	11	12	15	16	18	20

Table 1. The table shows the expected and actual diameters of the associahedra defined by triangulating an $(n+5)$-gon for small values of n.

One idea is to transpose all of this into the möbius strip of possible diagonal (viewed as off diagonal entries in a toris matrix). Once the flip rules are converted over and the positions corresponding to nonnegatively curved and fan triangulations are recognized it might be clear where a lower bound is coming from. Maybe I should make this a game on gap.

The shape you get when you match up the trees for $0^3(10)^3 1^3$ and $1^3(01)^3 0^3$ is a hexagonal antiprism.

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