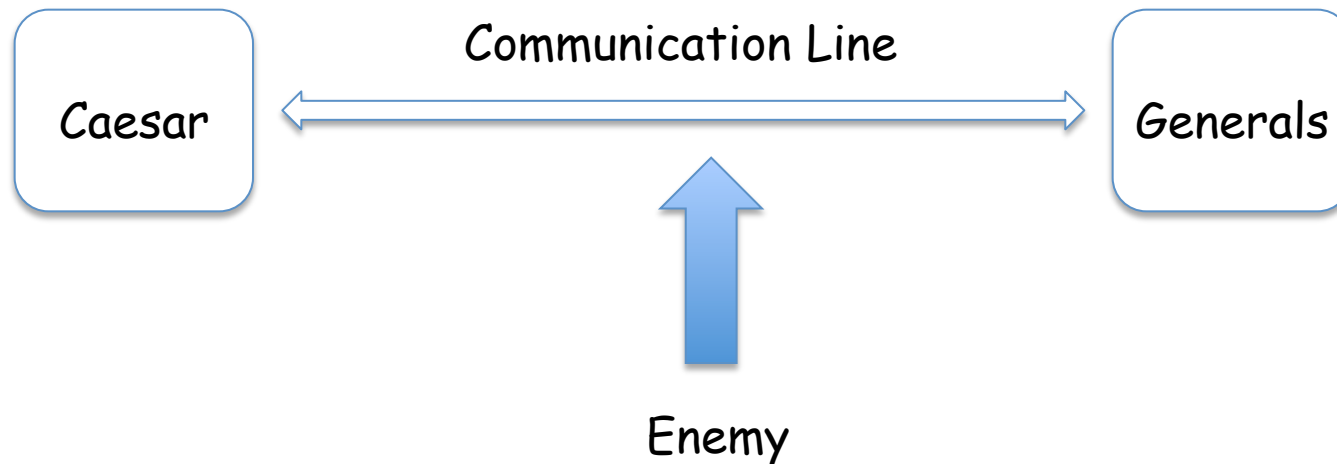


The War of Codemakers & Codebreakers

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Since the time of Caesar or even earlier, people are interested in "secret communications"



To hide the content of his messages from the enemy, Caesar developed "an encryption method" = **Caesar Cipher**

Caesar Cipher

For every letter in the word, replace the letter with the letter 3 locations ahead in the alphabet

a b c d e f g h i j k l m n o p q r s t u v w x y z

For example, if Caesar wants to send the order "attack", he encrypts it as:

attack -> **dwwdfn**

and sends it to his General.

The enemy also captures the message and sees "dwwdfn".
But the enemy does not know what that means!! 😊

The General decrypts "dwwdfn" by replacing every letter with the letter 3 locations back in the alphabet. 😊

dwwdfn -> **attack**

Encryption

A transformation of the message such that

- ✧ your enemy captures the encrypted message
 - ✧ your enemy should not be able to decrypt
 - ✧ your friend receives the encrypted message
 - ✧ your friend decrypts and obtains the message
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- ✧ cryptography: science of making encryption methods
 - ✧ cryptanalysis: science of breaking encryption methods
 - ✧ cryptology: cryptography + cryptanalysis
 - ✧ to encrypt, to decrypt; to encipher, to decipher
 - ✧ cipher: cryptographic (crypto) algorithm
 - ✧ message: meaningful text you are sending
 - ✧ ciphertext: encrypted text
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- ✧ There is "a civilized war" between cryptographers & cryptanalysts ... codemakers & codebreakers

Exercises:

1. Encrypt "wait" using Caesar cipher
2. Encrypt "yield" using Caesar cipher
3. Encrypt "return now" using Caesar cipher
4. Decrypt "uxq iru brxu olih" using Caesar cipher

Representation

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

q	r	s	t	u	v	w	x	y	z
16	17	18	19	20	21	22	23	24	25

Every letter is represented as a number between 0 and 25
Instead of working with letters we work with numbers

Affine Cipher

Affine cipher encrypts or decrypts a number-represented letter using the formula

Encrypt using $\alpha = \alpha + k \pmod{26}$

Decrypt using $\alpha = \alpha - k \pmod{26}$

Here k is known to you and your friend
The enemy does not (should not) know k

k is called the secret key

Caesar cipher is an affine Cipher with $k = 3$

Encrypt using $\alpha = \alpha + 3 \pmod{26}$

Decrypt using $\alpha = \alpha - 3 \pmod{26}$

Affine Cipher Example

$k = 11$

Represent "dinner" using numbers "3 8 13 13 4 17"

Then encrypt "dinner" = "3 8 13 13 4 17" using $k=11$

$$3 + 11 = 14 \pmod{26} \quad \rightarrow o$$

$$8 + 11 = 19 \pmod{26} \quad \rightarrow t$$

$$13 + 11 = 24 \pmod{26} \quad \rightarrow y$$

$$13 + 11 = 24 \pmod{26} \quad \rightarrow y$$

$$4 + 11 = 15 \pmod{26} \quad \rightarrow p$$

$$17 + 11 = 28 = 2 \pmod{26} \quad \rightarrow c$$

"dinner" is encrypted as "otyypc"

Affine Cipher

Decryption of "otyypc" = "14 19 24 24 15 2"

Decryption method: $\alpha = \alpha - k \pmod{26}$

$$14 - 11 = 3 \pmod{26} \quad \rightarrow d$$

$$19 - 11 = 8 \pmod{26} \quad \rightarrow i$$

$$24 - 11 = 13 \pmod{26} \quad \rightarrow n$$

$$24 - 11 = 13 \pmod{26} \quad \rightarrow n$$

$$15 - 11 = 4 \pmod{26} \quad \rightarrow e$$

$$2 - 11 = -9 = 17 \pmod{26} \quad \rightarrow r$$

The decrypted text is "dinner"

Exercises:

5. Encrypt "avatar" using the affine cipher with $k=0$

6. Encrypt "rain" using the affine cipher with $k=10$

7. Decrypt "wtaad" using the affine cipher with $k=15$

8. Decrypt "cxeeh" using affine cipher with $k=19$

Breaking Ciphers!!! 😊

Breaking or cryptanalysis of a cipher means

- ✧ either: decrypting without knowing the unknown key
- ✧ or: discovering the unknown key

Method 1: Try all possible keys

This encrypted message is given: "httpnj" and we don't know k (key) ... i.e., we are the enemy!! 😊

"httpnj" = "7 19 19 15 13 9"

Remember, decryption rule: $\alpha = \alpha - k \pmod{26}$

But we don't know k ... so we will try all possible values for it

All possible values of k are 0, 1, 2, 3, ..., 25

PS: No need to try k = 0, since k = 0 doesn't hide the message

Method 1: Try all possible keys

"httpnj" = "7 19 19 15 13 9"

Decryption rule : $\alpha = \alpha - k \pmod{26}$

Try $k = 1, 2, 3, \dots, 25$

$k=1$... "6 18 18 14 12 8" = "gssomi" ??

$k=2$... "5 17 17 13 11 7" = "frrnlh" ??

$k=3$... "4 16 16 12 10 6" = "eqqmkg" ??

$k=4$... "3 15 15 11 9 5" = "dppljf" ??

$k=5$... "2 14 14 10 8 4" = "cookie" 😊

$k=6$... "1 13 13 9 7 3" = "bnnjhd" ??

$k=7$... "0 12 12 8 6 2" = "ammigc" ??

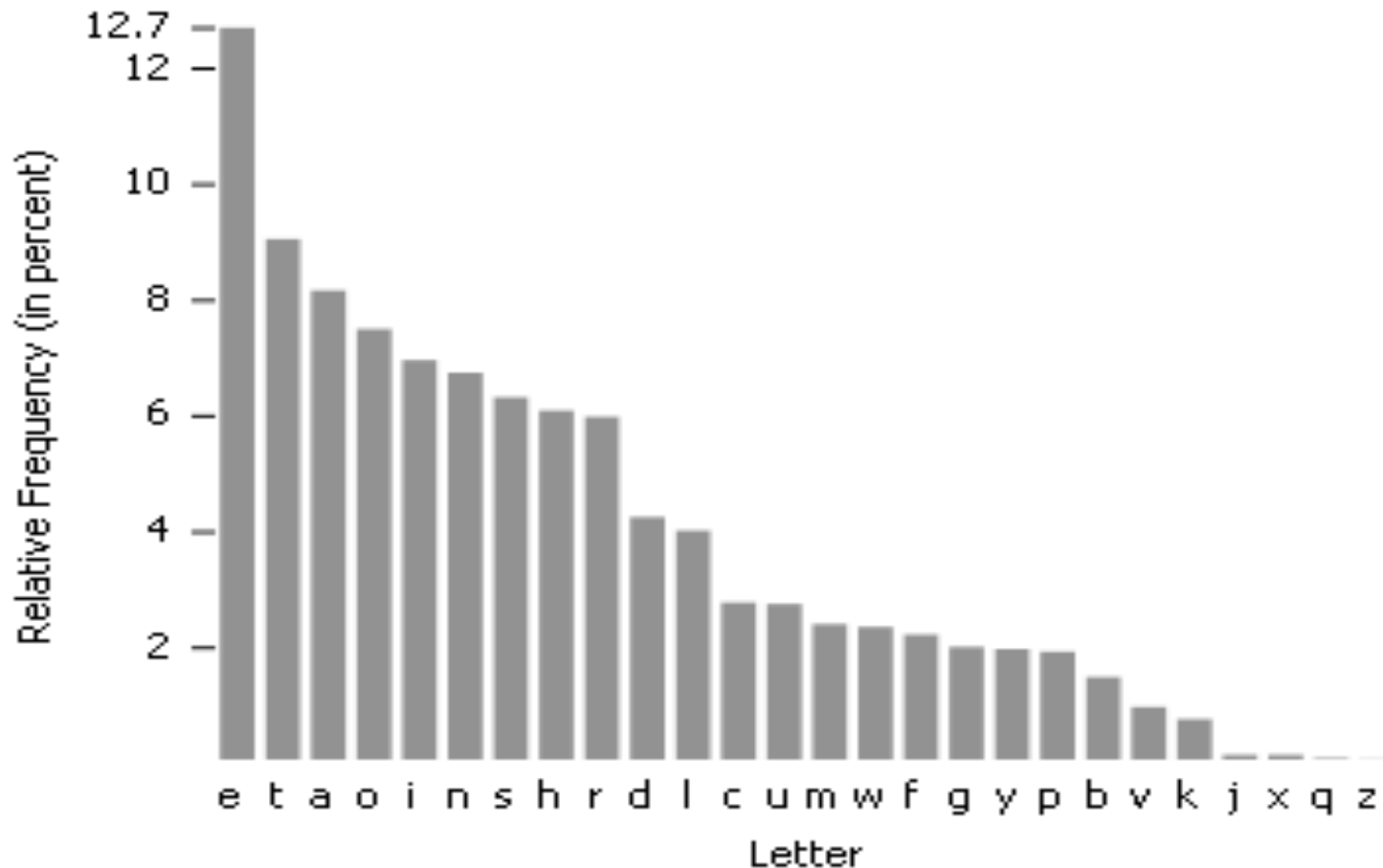
$k=8$... "25 11 11 7 5 1" = "zllhfb" ??

...

Fortunately, there are only 25 keys ... 😊

Method 2: Frequency of Letters

In an arbitrary English text, some letters appear more often than others: letter e appears the most, then letter t, then letter a, ... , letter z appears the least



Method 2: Frequency of Letters

Suppose the following encrypted sentence is given

"tbxqebo fp dobxq ebob"

and we are trying to find the key

In the encrypted text, the letter "b" appears the most often!!

Very likely "b" is the encryption of "e"

$$\text{"b"} = \text{"e"} + k \pmod{26}$$

$$1 = 4 + k \pmod{26}$$

This means $k = 23$ because $4 + 23 = 27 = 1 \pmod{26}$.

Using $k = 23$ in the above text, we decrypt it at once:

"tbxqebo fp dnbxq ebnb" -> "weather is great here"

Method 2 is better ... we did not have to try all possible keys 😊

Exercises:

9. Break the encryption of "xurq ue nqmgfurgx"

10. Break the encryption of "vhytqoi qhu jxu ruij"

11. Break the encryption of "tboub cbscbsb jt b gvo upxo"

12. Break the encryption of "cn hypyl luchm ch wufczilhcu"