HOMEWORK 2

8 PROBLEMS
DUE: WEDNESDAY, FEBRUARY 2, 2011

(1) Let G be a simple graph where the vertices correspond to each of the squares of an 8×8 chess board and where two squares are adjacent if, and only if, a knight can go from one square to the other in one move. What is/are the possible degree(s) of a vertex in G? How many vertices have each degree? How many edges does G have?

(2) Let G be a graph with n vertices and exactly $n-1$ edges. Prove that G has either a vertex of degree 1 or an isolated vertex.

(3) Prove that if a graph G has exactly two vertices u and v of odd degree, then G has a u, v-path.

(4) Let G be a simple graph. Show that either G or its complement $\overline{G}$ is connected.

(5) Are any of the graphs N_n, P_n, C_n, K_n and $K_{n,n}$ complements of each other?

(6) Show that if a simple graph G is isomorphic to its complement $\overline{G}$, then G has either $4k$ or $4k+1$ vertices for some natural number k. Find all simple graphs on four and five vertices that are isomorphic to their complements.

(7) The complete bipartite graphs $K_{1,n}$, known as the **star graphs**, are trees. Prove that the star graphs are the only complete bipartite graphs which are trees.

(8) A graph G is bipartite if there exists nonempty sets X and Y such that $V(G) = X \cup Y$, $X \cap Y = \emptyset$ and each edge in G has one endvertex in X and one endvertex in Y. Prove that any tree with at least two vertices is a bipartite graph.