HOMEWORK 3

8 PROBLEMS
DUE: WEDNESDAY, FEBRUARY 16, 2011

(1) Show that for each $n \in \mathbb{N}$ the complete graph K_n is a contraction of $K_{n,n}$.

(2) For $n \in \mathbb{N}$, can K_n be a contraction of $K_{m,n}$ if $m < n$?

(3) The complete tripartite graph $K_{r,s,t}$ consists of three disjoint sets of vertices (of sizes r, s and t), with an edge joining two vertices if and only if they lie in different sets. Draw $K_{2,2,2}$. What is the number of edges of $K_{2,3,4}$?

(4) There are exactly 11 unlabeled trees on seven vertices. Draw these eleven trees, making sure that no two are isomorphic.

(5) Show that every tree containing a vertex of degree k contains at least k leaves.

(6) For two points in $\mathbb{R}^2$, $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, let $d : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be given by

$$d(P_1, P_2) = |x_2 - x_1| + |y_2 - y_1|.$$

Show that d is a metric on $\mathbb{R}^2$.

(7) For all $n \in N$ what is the eccentricity of each vertex of K_n? How many centers does K_n have?

(8) Draw all spanning trees of the graph G.

G :