HOMEWORK 4

8 PROBLEMS
DUE: WEDNESDAY, MARCH 2, 2011

(1) Draw the tree whose Prüfer code is $(1,1,1,1,6,5)$.

(2) Determine which trees have Prüfer codes that have distinct values in all positions.

(3) Let G be a connected graph which is not a tree and let C be a cycle in G. Prove that the complement of any spanning tree of G contains at least one edge of C.

(4) Suppose a graph G is formed by taking two disjoint connected graphs G_1 and G_2 and identifying a vertex in G_1 with a vertex in G_2. Show that $\tau(G) = \tau(G_1)\tau(G_2)$.

(5) Assume the graph G has two components G_1 and G_2. Show there is a labeling of the vertices of G such that the adjacency matrix of G has the form

$$\mathbf{A}(G) = \begin{pmatrix} \mathbf{A}(G_1) & \mathbf{0} \\ \mathbf{0} & \mathbf{A}(G_2) \end{pmatrix}.$$

(6) An *m-fold path*, mP_n, is formed from P_n by replacing each edge with a multiple edge of multiplicity m. An *m-fold cycle*, mC_n, is formed from C_n by replacing each edge with a multiple edge of multiplicity m.

  (a) Find $\tau(mP_n)$
  (b) Find $\tau(mC_n)$

(7) Find $\tau(K_{2,3})$.

(8) Use the Matrix-Tree Formula to compute $\tau(K_{3,n})$.