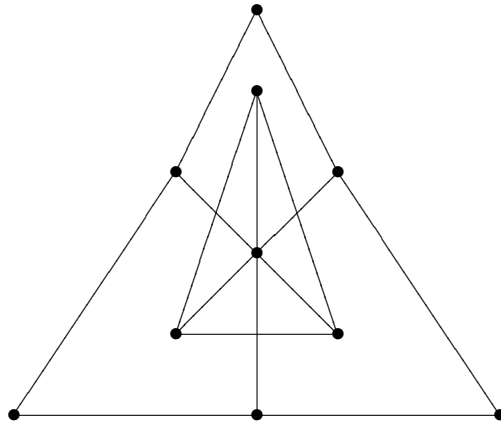


### HOMEWORK 3

8 PROBLEMS  
DUE: WEDNESDAY, MAY 11, 2011

- (1) Show that the graph given below is planar.



- (2) Let  $G$  be a plane graph. If  $e \in G$ , what is the plane dual of the contraction  $(G \cdot e)^*$  in terms of the plane dual  $G^*$  of  $G$ ?
- (3) Let  $G$  be a simple connected graph with at least 11 vertices. Prove that either  $G$  or its complement  $\overline{G}$  must be nonplanar.
- (4) Let  $G$  be a simple connected planar graph with less than 12 vertices. Prove that  $G$  has a vertex of degree at most 4.
- (5) Let  $G$  be a simple connected planar graph with less than 30 edges. Prove that  $G$  has a vertex of degree at most 4.
- (6) Show that  $K_7$  is toroidal.
- (7) Prove that  $K_n$  is toroidal if, and only if  $n \in \{5, 6, 7\}$ .
- (8) What is the maximum number of edges in a graph with  $n$  vertices and genus  $\gamma$ ? Justify, your answer.