MATH 137B Midterm

Tuesday, May 10, 2011

Solutions

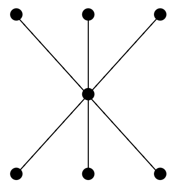
1. Show that two 1-isomorphic graphs have the same number of edges.

   Solution: Let G and G' be 1-isomorphic graphs. By definition, both graphs can be reduced, by successive 1-split operations, to the same graph H, where each component of H is a block of G (and G'), and vice versa.

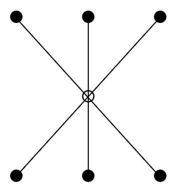
   Since a 1-split operation neither adds nor subtracts an edge, we have $E(G) = E(H)$ and $E(G') = E(H)$. Hence, $E(G) = E(G')$.

2. Find a graph on seven vertices that is isomorphic to its block-cutpoint graph.

   Solution: Consider the star graph $K_{1,6}$

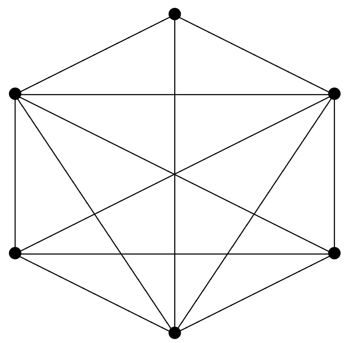
   

   The center vertex is the only cutpoint and each edge is a block of $K_{1,6}$. Hence, $\mathrm{BC}(K_{1,6})$ is the graph

   which is clearly isomorphic to $K_{1,6}$.

3. Determine whether or not the graph given below is planar.



Justify your answer in *two different* ways.

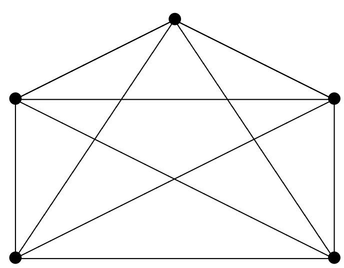
Solution:

(i) By Corollary 7.15 of the text, if G is a simple planar graph on $n \geq 3$ vertices with m edges then

$$m \leq 3n - 6.$$

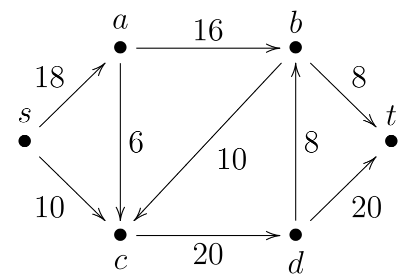
As the given graph has 6 vertices and 13 edges and $13 > 3 \times 6 - 6 = 12$, it is not planar.

(ii) Performing a simple contraction on the long edge that runs north to south gives a K_5 minor.



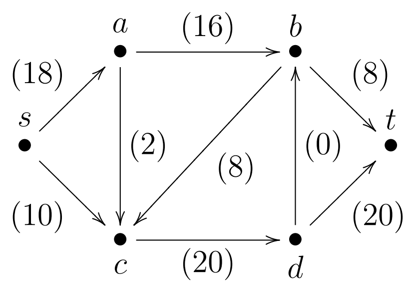
Thus, by Wagner's Theorem, the given graph is not planar.

4. Consider the network given below.



Find a maximum flow. Justify your answer.

Solution: Consider the indicated function on the set of edges of the network graph.



This function is indeed a flow, as for each edge the function value is less than or equal to capacity and for each vertex (except for the source and sink) the Kirchhoff conditions holds.

The flow is maximum: Since the flow is equal to capacity for all edges adjacent to the source, there can be no augmenting path.