Math 3A: Calculus with Applications I
Final Paper

In lieu of a final exam, you will be asked to write a final paper for this course.

Why a final paper?

There are a number of reasons why you are being asked to complete an essay assignment in this course.

- If you continue to study mathematics, or another discipline which uses mathematics as a significant tool, then you will increasingly be asked to write about mathematical concepts. Mathematical writing takes a different guise than writing in other disciplines, so it is important to practice communicating effectively.
- You have already been given a number of timed assignments during the course in the form of quizzes and midterm exams. This essay is an opportunity for you to demonstrate your understanding of the course material in a non-timed, low-stakes environment.

Prompt:

You will choose a significant topic from this course and write an expository paper on this topic. Some possible topics are listed below, but you are welcome to to find another appropriate topic. If you are unsure about if a topic would be appropriate, feel free to ask first! This paper should summarize the main definitions and theorems relating to this concept and include a variety of examples which clearly illustrate this concept in practice. You may want to include formal proofs of any significant results. Your paper should discuss connections between this concept and other concepts in calculus or other mathematical disciplines. Your paper should argue for the inherent benefit of this concept and/or discuss the connections to real-world applications or other disciplines. At the bottom of this assignment sheet there are a number of questions for you to think about when writing your paper.

In Summary:

I'd like you to pick a key topic from the course and write extensively about it. Your paper should include:

- A detailed explanation of the topic. You should explain the topic intuitively as well as rigorously. This will possibly include a proof of the topic. Once you have chosen a specific topic, I am happy to talk in more detail about what proofs (if any) might be helpful here. Depending on the topic, an informal proof might be sufficient.
- Justification for why this topic matters. You should talk about how your topic relates to other important topics from the course and discuss any practical applications and/or its applications to other parts of math or other academic disciplines. This would be an excellent opportunity for you to research how your chosen topic connects to your major-field, for instance.
- Several detailed examples which illustrate the full range of the topic and how it is used in practice. These examples should be varied–have you included different types of functions? Have you demonstrated all of the possible applications of your chosen topic?

- Relevant diagrams or graphs. These diagrams should supplement both your exposition of the topic and your examples and should be seamlessly integrated into your writing.
- A references section, with appropriate in-text citations as needed.

You can also read through the attached rubric to get a better sense of what I am looking for in your paper. Note that some portions of the rubric are worth twice as many points as other sections.

A few possible paper topics:

- Related rates
- Optimization
- Linear approximation and Newton's method
- The mean value theorem and its applications
- Some other topic–feel free to run your ideas by me first to make sure it is appropriate.

Logistics:

You may use any resources that you like, but please cite your sources appropriately. There is no required page length for this paper, but probably somewhere from $5-10$ pages typed is reasonable. You should focus more on including all of the required material than on the page length. The paper is due the Saturday after the last day of class. You will be graded using the attached rubric.

Typesetting information:

Your final paper should be typed, preferably using LaTeX. Additional resources for getting started with typesetting using LaTeX are posted on the GauchoSpace page. LaTeX is the default typesetting program for writing in mathematics as well as a range of other disciplines, so being able to use it effectively is a great skill to have–especially as you begin taking upper division courses. In addition to editing the content of your paper before turning it in, you should also make sure that the document is typeset in a readable fashion before turning it in. This includes making sure that all of your images and mathematical notation are showing up correctly. Do not leave this until the last minute, especially if you have never used LaTeX before.

A few preliminary style guidelines:

- You should follow all of the standard rules of grammar and syntax that you would for an essay in a humanities discipline. Write clearly and use mathematically appropriate language.
- Your target audience for this paper is someone with a similar math background as yourself. Provide as much detail as you wish someone had given you when you first learned about this topic. You might imagine that you are writing for a confused student in the course.
- Before you include any computations, write a sentence or two to explain what you are about to do. After you finish a long computation or proof, review what this has actually shown.
- Clearly define any functions and variables that you use. For example, a sentence such as "The area of a circle is πr^2," is not particularly clear, as it has not defined the value r. Instead, write something like "The area of a circle of radius r is πr^2."
- Because much of mathematics relies on formal definitions, when writing an expository paper in math it is easy to accidentally plagiarize someone else's work. To avoid this, you should write your paper with "the book" closed. Read as many sources as you'd like to get comfortable with the material, but do your writing without looking at them to make sure you are using your own words. Copying text,

proofs, or computations verbatim from the text is not acceptable. If you use diagrams from the text or other sources, make sure to cite them appropriately.

- A number of resources for writing in mathematics have been posted to the GauchoSpace page. Try reading through these to get a better feel for what good writing in mathematics looks like.

A checklist to think about when writing:

For more information on how your paper will be graded, please see the included rubric. However, these are some questions that you might want to think about and answer as you write your paper.

- Have I clearly introduced the main topic as well as any related definitions and theorems? Have I provided a proof of this concept–either formal or informal?
- Is my exposition of the main topic clear? If someone were to only read this paper, would they be able to understand this concept clearly?
- Have I used appropriate diagrams to help illustrate this concept?
- Have I explained why we care about this concept and how it is used in calculus?
- Have I discussed how this topic relates to other topics from this course? To other topics in mathematics? To other disciplines outside of mathematics?
- Have I included a variety of examples which clearly illustrate the full range of implementations and applications of this concept?

Math 3A: Grading Rubric for Final Paper

Grading Criteria	**Grading Scale and Score** ($\times 2$)			
	Exemplary (4)	**Meets Expectations (3)**	**Progressing (2)**	**No Evidence to Support (1)**
Use of Mathematical Language	Exemplary work uses precise and illuminating mathematical language and notation throughout. All notation is clearly introduced and definitions are consistent throughout the paper.	Expected work uses correct mathematical language, but some phrasing or notation could be improved. There may be some minor inconsistencies which don't impact the content of the paper.	Progressing work shows a developing understanding of appropriate mathematical language, but there are significant	This work shows minimal understanding of the related mathematical language. Definitions are incorrect or not included.
Definitions and Theorems	Exemplary work clearly defines any related concepts or theorems, which allows the reader to follow the paper with ease. Definitions are generally included at the level of a 3A student, and an intuitive explanation of definitions are included where helpful.	Expected work defines most important concepts, but may not define one or two important objects, or an occasional definition might not be at the appropriate level for a 3A student.	Progressing work has an inconsistent use of definitions throughout. There might be a significant number of mathematical objects left undefined, or the majority of definitions are not written at an appropriate level.	This work includes few clear and appropriate definitions of the related mathematical concepts.
Graphs, diagrams, and Figures	Exemplary work includes graphs, diagrams, and figures throughout to help clearly convey the concepts at hand. These diagrams or figures augment the material, and it is obvious how they integrate into the written work.	Expected work includes graphs, diagrams, and figures, but there may be occasional instances where a diagram would clarify the exposition but is not included.	Progressing work includes figures, but there are several instances where a diagram would clarify the exposition but none is included. The figures may not be clearly integrated into the written text.	This work includes few or no figures, and/or it is not apparent how figures that are included integrate into the text.

Grading Criteria	**Grading Scale and Score** ($\times 2$)			
	Exemplary (4)	**Meets Expectations (3)**	**Progressing (2)**	**No Evidence to Support (1)**
Connections	Exemplary work clearly connects the chosen topic to other topics in the course, other areas of mathematics, and other disciplines.	Expected work connects the chosen topic to some of these areas, but may lack the depth of exemplary work.	Progressing work makes some connections to other topics from the course, other areas of math, or other disciplines, but these connections are brief and/or surface level.	This work includes few or no meaningful connections to other subject areas.
Proof of Main Topic	Exemplary work includes a clear informal proof or justification of the main topic, written in the student's own words. Depending on the topic, exemplary work also includes a formal proof of the topic or of related concepts or theorems.	Expected work includes a clear proof or justification of the topic, but might lack some of the depth or details of exemplary work.	Progressing work shows a developing understanding of the intuition behind the topic, but does not include a sufficiently clear justification or proof.	This work shows minimal understanding of the rationale behind the main topic. A proof is either incorrect or not included.
Motivation	Exemplary work clearly motivates the topic and illustrates the value of the concept at hand. Exemplary work makes it evident why the chosen topic has intrinsic value in calculus.	Expected work includes some justification, but lacks the depth and clarity of exemplary work.	Progressing work provides some justification, but this justification is minimal or narrow in its scope.	This work includes little to no justification for why this is an important topic in calculus.
Depth of mathematical content	Exemplary work demonstrates a thorough understanding the material by communicating the value of the ideas and making important connections to other concepts, areas of mathematics, or other disciplines. Concepts are explained intuitively and clear proofs or proof sketches are included where they would enhance understanding.	Expected work demonstrates a clear understanding of the material, but might miss some connections to other material. The exposition is generally good, but some parts could be edited or expanded for increased clarity.	Progressing work shows a basic understanding of the course content. This work includes basic exposition, but doesn't make sufficient connections to other work and/or lacks formal or informal proof.	This work shows little to no understanding of the mathematical content. There are significant errors in the mathematical material or major concepts are entirely ignored.

Grading Criteria	**Grading Scale and Score ($\times 2$)**			
	Exemplary (4)	**Meets Expectations (3)**	**Progressing (2)**	**No Evidence to Support (1)**
Use of Examples	Exemplary work includes a variety of examples which demonstrate the full range of the concept at hand. These examples connect the main topic to other topics discussed in the course. Graphs and diagrams are used seamlessly to augment the exposition.	Expected work uses examples which highlight the main concepts, but may include some repetitive or unenlightening examples. Some of these examples connect to other content in the course, but the connections might not be as thoroughly intertwined as in exemplary work.	Progressing work uses some examples, but they are minimal and not particularly varied. Connections to other work and diagrams are minimal if existent.	This work includes no examples or the examples included are trivial and non-illustrative.
Integration of Math and Exposition	Exemplary work clearly integrates written explanations and justifications into any examples or calculations. Variables are clearly defined and introduced in the text, and written sentences accompany any algebraic steps. Exemplary work is easy to follow–it is obvious when reading this work what mathematical steps are being taken and why.	Expected work is similar to exemplary work, but lacks the same depth and clarity. There might be occasional instances where steps are not justified, or additional text would help clarify the mathematical work.	Progressing work shows an inconsistent use of justifications throughout the paper. There might be occasional examples with long strings of equations without written explanation or occasional variables which are not explicitly defined.	This work includes little to no written explanations accompanying any algebraic work. Examples are difficult to comprehend and require a significant effort on the part of the reader to follow them.

Grading Criteria	**Grading Scale and Score**			
	Exemplary (4)	**Meets Expectations (3)**	**Progressing (2)**	**No Evidence to Support (1)**
Writing style and structure	Exemplary work is well-organized, cohesive, and flows in a logical manner. An exemplary paper has a distinctive voice, transitions seamlessly between topics, and is entertaining to read.	Expected work is coherent, well-organized, and logically ordered. There may be some minor room for improvement with regards to structure and organization.	Progressing work is readable but has some jarring transitions or style. There may be significant errors in cohesion or sentence structure.	This work is poorly organized and difficult to read. Little to no thought has been given to the structure of the paper and there is no distinctive voice throughout.
Editing and layout	Exemplary work shows use of proper grammar and syntax and appropriately cites all sources. The typesetting is readable and consistent throughout. All diagrams are easy to read and clearly labelled.	Expected work may have a few minor errors in grammar or syntax which don't majorly impact the content of the paper. Diagrams are readable but could maybe be more seamlessly embedded into the paper.	Progressing work has more frequent errors in grammar or syntax which might have some impact on the readability of the paper. Diagrams might be difficult to read or not clearly labelled.	This work has serious errors in grammar or syntax which make it difficult to read the paper. This paper might not include any diagrams, or those diagrams might be illegible or entirely unlabelled.
Readable Use of LaTeX	Exemplary work has a readable use of LaTeX throughout. Equations and mathematical notation are always in math-mode and the correct commands for typesetting math are used.	Expected work may have a few minor errors in its use of LaTeX which don't majorly impact the content of the paper.	Progressing work has more frequent errors in its use of LaTeX which might have some impact on the readability of the paper.	This work has serious errors in its use of LaTeX which make it difficult to read the paper.