UCSB: MATH 8
SECTION THREE
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Name:

Problem 1

Analyze the logical forms of the following statements:

(a) 3 is a common divisor of 6, 9, and 15.

(b) x is divisible by both 2 and 3 but not 4.

(c) x and y are natural numbers, and exactly one of them is prime.

Solution

Problem 2

Write definitions using elementhood tests for the following sets:

(a) $\{1, 4, 9, 16, 25, 36, 49, \ldots\}$.

(b) $\{1, 2, 4, 8, 16, 32, 64, \ldots\}$.

(c) $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$.

Solution

Problem 3

What do the following statements mean? Your answer should be a simple English sentence.

(a) $a + b \notin \{x \mid x \text{ is an even number}\}$.

(b) $y \in \{x : x \text{ is divisible by } w\}$.

(c) $4 \in \{w \mid 6 \notin \{x \mid x \text{ is divisible by } w\}\}$.

Solution

Problem 4

For each of the following sets, write out (using logical symbols) what it means for an object x to be an element of the set. Then determine which of these sets must be equal to each other by determining which statements are equivalent.

(a) $(A \setminus B) \setminus C$.

(b) $A \setminus (B \setminus C)$.

(c) $(A \setminus B) \cup (A \cap C)$.

(d) $(A \setminus B) \cap A \setminus C)$.

(e) $A \setminus (B \cup C)$.

Solution

Problem 5

Verify the following identities by writing out (using logical symbols) what it means for an object x to be an element of each set and then using logical equivalences:

(a) $A \setminus (A \cap B) = A \setminus B$.

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(c) $A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A)$.

Solution

Problem 6

Use Venn diagrams to verify the same identities as in the problem above.

Solution