Problem 1

Use truth tables to determine whether or not the following arguments are valid:

(a) If the tax rate and the unemployment rate both go up, then there will be a recession. If teh GNP goes up, then there will not be a recession. The GNP and taxes are both going up. Therefore, the unemployment rate is not going up.

(b) The warning light will come on if and only if the pressure is too high and the relief valve is clogged. The relief valve is not clogged. Therefore, the warning light will come on if and only if the pressure is too high.

Solution

Problem 2

(a) Show that $P \leftrightarrow Q$ is equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.

(b) Show that $(P \rightarrow Q) \vee (P \rightarrow R)$ is equivalent to $P \rightarrow (Q \vee R)$.

(c) Show that $(P \rightarrow R) \wedge (Q \rightarrow R)$ is equivalent to $(P \vee Q) \rightarrow R$.

(d) Formulate and verify a similar equivalence to part (c) involving $(P \rightarrow R) \vee (Q \rightarrow R)$.

Solution

Problem 3

(a) Show that $(P \to Q) \wedge (Q \to R)$ is equivalent to $(P \to R) \wedge [(P \leftrightarrow Q) \vee (R \leftrightarrow Q)]$.

(b) Show that $(P \to Q) \vee (Q \to R)$ is a tautology.

(c) Find a formula involving only the connectives $\neg$ and $\to$ that is equivalent to $P \wedge Q$.

(d) Find a formula involving only the connectives $\neg$ and $\to$ that is equivalent to $P \leftrightarrow Q$.

Solution

Problem 4

Which of the following formulas are equivalent?

(a) $P \to (Q \to R)$.

(b) $Q \to (P \to R)$.

(c) $(P \to Q) \wedge (P \to R)$.

(d) $(P \wedge Q) \to R$.

(e) $P \to (Q \wedge R)$.

Solution