UCSB: Math 8

Section Five

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Name:

Problem 1

What do the following statements mean? Are they true or false? The universe of discourse in each case is $\mathbb{N}$, the set of all natural numbers.

(a) $\forall x \exists y (x < y)$

(b) $\exists y \forall x (x < y)$

(c) $\exists x \forall y (x < y)$

(d) $\forall y \exists x (x < y)$

(e) $\exists x \exists y (x < y)$

(f) $\forall x \forall y (x < y)$

Solution

Problem 2

Analyze the logical forms of the following statements The universe of discourse is $\mathbb{R}$. What are the free variables in each statement?

(a) Every number that is larger than x is larger than y.

(b) For every number a, the equation $ax^2 + 4x - 2 = 0$ has at least one solution if and only if $a \geq -2$.

(c) All solutions of the inequality $x^3 - 3x < 3$ are smaller than 10.

(d) If there is a number x such that $x^2 + 5x = w$ and there is a number y such that $4 - y^2 = w$, then w is between -10 and 10.

Solution

Problem 3

Are these statements true or false? The universe of discourse is $\mathbb{N}$.

(a) $\forall x \exists y (2x - y = 0)$.

(b) $\exists y \forall x (2x - y = 0)$.

(c) $\forall x \exists y (x - 2y = 0)$.

(d) $\forall x (x < 10 \rightarrow \forall y (y < x \rightarrow y < 9))$.

(e) $\exists y \exists z (y + z = 100))$.

(f) $\forall x \exists y (y > x \wedge \exists z (y + z = 100))$.

Solution

Problem 4

Repeat the above exercise, but with:

(a) $\mathbb{R}$ as the universe of discourse.

(b) $\mathbb{Z}$ as the universe of discourse.

Solution