UCSB: MATH 8
SECTION SIX
TA: SETH ALTHAUSER
EMAIL: ALTHAUSER@MATH.UCSB.EDU

Name:

Problem 1

(a) Show that the existential quantifier distributes over disjunction. In other words, show that $\exists x(P(x) \vee Q(x))$ is equivalent to $\exists x P(x) \vee \exists x Q(x)$.

(b) Is $\forall x(P(x) \vee Q(x))$ equivalent to $\forall x P(x) \vee \forall x Q(x)$? Explain. (Hint: Try assigning meanings to $P(x)$ and $Q(x)$).

Solution

Problem 2

(a) Show that $(\forall x \in A, P(x)) \wedge (\forall x \in B, P(x))$ is equivalent to $\forall x \in (A \cup B), P(x)$.

(b) Show that $\exists x \in A, P(x) \vee \exists x \in B, P(x)$ is equivalent to $\exists x \in (A \cup B), P(x)$.

(c) Is $\exists x \in A, P(x) \wedge \exists x \in B, P(x)$ equivalent to $\exists x \in (A \cap B), P(x)$? Explain your answer.

Solution

Problem 3

Show that the statements $A \subseteq B$ and $A \setminus B = \emptyset$ are equivalent by writing each in logical symbols and then showing that the resulting formulas are equivalent.

Solution

Problem 4

Let $T(x, y)$ mean "x is a teacher of y." What do the following statements mean? Under what circumstances would each one be true? Area ny of them equivalent to each other?

(a) $\exists! y T(x, y)$.

(b) $\exists x \exists! y T(x, y)$.

(c) $\exists! x \exists y T(x, y)$.

(d) $\exists y \exists! x T(x, y)$.

(e) $\exists! x \exists! y T(x, y)$.

(f) $\exists x \exists y [T(x, y) \wedge \neg \exists u \exists v (T(u, v) \wedge (u \neq x \vee v \neq y))]$.

Solution