UCSB: MATH 8
SECTION SEVEN
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Name:

Problem 1

Suppose a, b, c and d are real numbers, $0 < a < b$, and $d > 0$. Prove that if $ac > bd$ then $c > d$.

Scratch Work

Solution

Problem 2

Suppose that a is a real number. Prove that if $a^3 > a$, then $a^5 > a$.

Scratch Work

Solution

Problem 3

Prove the following statement: if x is odd, then x^2 is odd.

Scratch Work

Solution

Problem 4

Suppose a and b are real numbers. Prove that if $a < b$, then $\frac{a+b}{2} < b$.

Scratch Work

Solution

Problem 5

Let x and y be positive real numbers. Prove that if $x \leq y$, then $\sqrt{x} \leq \sqrt{y}$.

Scratch Work

Solution

Problem 6

Prove that every odd integer is a difference of squares–i.e. show that an odd integer can be written as $x^2 - y^2$ for an appropriate choice of x and y.

Solution

Problem 7

(a) Let n and k be positive integers with $1 < k \leq n$. Prove that $n! + k$ is composite. (Thus for any $n \geq 2$, one can find n consecutive composite numbers. This means there are arbitrarily large "gaps" between prime numbers).

(b) Use part (a) to find 100 consecutive integers, all of which are composite.

Solution