UCSB: MATH 8
SECTION EIGHT
TA: SETH ALTHAUSER
EMAIL: ALTHAUSER@MATH.UCSB.EDU

Name:

Problem 1

This problem could be solved by using truth tables, but don't do it that way. Instead, use the methods for writing proofs discussed in chapter 3 of your text.

(a) Suppose $P \to Q$ and $R \to \neg Q$ are both true. Prove that $P \to \neg R$ is true.

(b) Suppose that P is true. Prove that $Q \to \neg(Q \to \neg P)$ is true.

Scratch Work

Solution

Problem 2

Suppose $A \subseteq C$, and B and C are disjoint. Prove that if $x \in A$, then $x \notin B$.

Scratch Work

Solution

Problem 3

Suppose that $A \setminus B$ is disjoint from C and $x \in A$. Prove that if $x \in C$ then $x \in B$.

Scratch Work

Solution

Problem 4

Suppose that a and b are nonzero real numbers. Prove that if $a < \frac{1}{a} < b < \frac{1}{b}$ then $a < -1$.

Scratch Work

Solution

Problem 5

Suppose that $x, y \in \mathbb{Z}$. Prove that if $x + y$ is even, then x and y have the same parity.

Scratch Work

Solution

Problem 6

(a) Suppose that $x \in \mathbb{Z}$. Prove that if $x^2 - 6x + 5$ is even, then x is odd.

(b) Prove that if $n \in \mathbb{Z}$, then $n^2 + 3n + 4$ is even.

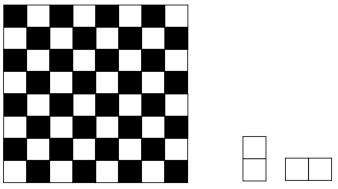
Scratch Work

Solution

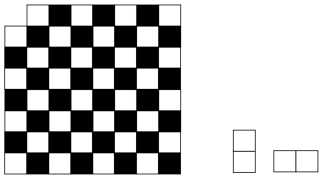
Problem 7

(A puzzle to think about–or maybe even prove!)

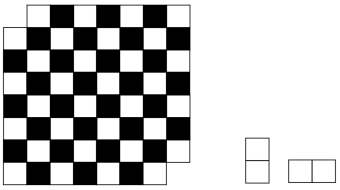
Can a checkerboard be tiled by 1×2 dominoes? (You can orient each domino as either a 1×2 rectangle or a 2×1 rectangle on the checkerboard).



What if we remove the top left corner from the board?



What if we remove both the top left corner and the bottom right corner?



What if we remove two arbitrary squares from the checkerboard?