UCSB: Math 8

Section Nine
TA: Seth Althauser
Email: althauser@math.ucsb.edu

Name:

Problem 1

Suppose that A, B, and C are sets and that $A \setminus B \subseteq C$. Prove that $A \setminus C \subseteq B$.

Scratch Work

Solution

Problem 2

(a) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(b) Prove that $A \cap (\bigcup_{i \in I} B_i) = \bigcup_{i \in I} (A \cap B_i)$.

Scratch Work

Solution

Problem 3

Prove that for every real number x, if $x > 0$ then there is a real number y such that $y(y+1) = x$.

Scratch Work

Solution

Problem 4

(a) Prove that for all real numbers x and y there is a real number z such that $x + z = y - z$.

(b) Would the statement in part (a) be correct if "real number" were changed to "integer?" Justify your answer.

Scratch Work

Solution

Problem 5

In this problem, all variables range over $\mathbb{Z}$.

(a) Prove that if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.

(b) Prove that if $ac \mid bc$ and $c \neq 0$, then $a \mid b$.

Scratch Work

Solution

Problem 6

Suppose $\{A_i \mid i \in I\}$ is an indexed family of sets. Prove that $\bigcup_{i \in I} \mathcal{P}(A_i) \subseteq \mathcal{P}(\bigcup_{i \in I} A_i)$. (Hint: First make sure you know what all the notation means!)

Scratch Work

Solution

Problem 7

Suppose that $\mathcal{F}$ and $\mathcal{G}$ are nonempty families of sets, and every element of $\mathcal{F}$ is a subset of every element of $\mathcal{G}$. Prove that $\bigcup \mathcal{F} \subseteq \bigcap \mathcal{G}$.

Scratch Work

Solution