Math 108A - Home Work # 2

Due: April 15, 2009

1. Problems 5, 13, 15 on p. 19-20 in LADR.

2. In class, we saw that the set $\mathcal{C}(\mathbb{R})$ of all continuous functions $f : \mathbb{R} \to \mathbb{R}$ is an $\mathbb{R}$-vector space (with the 0-function $0(x) = 0 \ \forall x \in \mathbb{R}$ as the 0-vector). Which of the following subsets of $\mathcal{C}(\mathbb{R})$ are subspaces? Justify your answers.

   (a) $\mathcal{C}^2(\mathbb{R}) = \{ f \in \mathcal{C}(\mathbb{R}) \mid f \text{ is twice differentiable } \}$

   (b) $\mathcal{E} = \{ f \in \mathcal{C}(\mathbb{R}) \mid f(0) = 1 \}$

   (c) $\mathcal{F} = \{ f \in \mathcal{C}(\mathbb{R}) \mid f(1) = 0 \}$

   (d) $\mathcal{G} = \{ f \in \mathcal{C}(\mathbb{R}) \mid \forall x \in \mathbb{R} \ f(x) \neq 0 \}$

   (e) $\mathcal{B} = \{ f \in \mathcal{C}(\mathbb{R}) \mid \exists M \in \mathbb{R} \ \forall x \in \mathbb{R} \ |f(x)| \leq M \}$ (The set of all bounded continuous functions.)

3. Recall the definition of the intersection of a family of sets indexed by a set I: If A_i is a set for each $i \in I$, then

$$\bigcap_{i \in I} A_i = \{ x \mid x \in A_i \forall i \in I \}.$$

   Suppose that V is a vector space over F, and suppose that V_i is a subspace of V for each $i \in I$. Show that the intersection $\bigcap_{\imath \in I} V_i$ is also a subspace of V.

4. **Extra Credit:** If U is any *subset* of a vector space V, we defined $span(U)$ as the set of linear combinations of elements of U, i.e.,

$$span(U) = \{ c_1 u_1 + \cdots + c_n u_n \mid \forall i \ c_i \in F, u_i \in V \},$$

   and we showed that $span(U)$ is a subspace of V. Show that $span(U)$ equals the intersection of all subspaces of V that contain the set U. (By the previous exercise, this gives another way of seeing that $span(U)$ is a subspace. We can also interpret this result as saying that $span(U)$ is the smallest subspace of V that contains U.)