Math 108A - Home Work # 4

Due: April 29, 2009

1. Exercises 13, 14 on p. 36 in LADR.

2. Consider the subspace $U = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + w = y + z\}$ in $\mathbb{R}^4$.

   (a) Show that $\mathbb{R}^4 = U \oplus \mathbb{R}(0, 0, 0, 1)$.

   (b) What is $\dim U$? (Suggestion: use (a).)

   (c) Find a basis for U, and justify why it is a basis (Part (b) is helpful).

3. Prove or give a counterexample: If $\{v_1, \ldots, v_n\}$ is any linearly dependent set of vectors, then for all i, v_i is a linear combination of the other vectors in the set.

4. Prove that $\{v_1, \ldots, v_m\}$ is a linearly independent set of vectors if and only if any $u \in span(v_1, \ldots, v_m)$ can be written uniquely as a linear combination $u = c_1 v_1 + \cdots + c_m v_m$ for scalars $c_1, \ldots, c_m \in F$.