Math 108A - Home Work # 8

Due: June 3, 2009

1. LADR, p. 94-5: Exercises

   - 5,
   - 7 (Don't try to find the characteristic polynomial. Instead, start by finding the kernel.),
   - 8 (You'll need to use the definition of Eigenvalues/Eigenvectors),
   - 10,
   - 11,
   - 12 (First, show that T can have only one Eigenvalue.).

2. As in Ex. 7, consider the matrix ($n = 3$)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

   (a) Find a change of basis matrix C such that $C^{-1}AC$ is diagonal. What is this diagonal matrix?

   (b) Compute A^{100}.

3. **Extra Credit:** The matrix $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ has the property that $A^2 = -I$. Find all 2×2 matrices B with this property (i.e., $B^2 = -I$). Hint: think about the eigenvalues of B.

4. Suppose that an $n \times n$ matrix B is diagonalizable, with 0 and 1 as its only eigenvalues. Show that $B^2 = B$. Is the converse true: i.e., if B is diagonalizable and $B^2 = B$, are 0 and 1 the only possible eigenvalues of B?