Solution

Math 108A - Quiz # 4

June 1, 2009

Let V be a vector space over F, not necessarily finite-dimensional.

1. What are the eigenvalues of the Identity Map $I_V : V \to V$?

$I_V(\vec{v}) = \vec{v} = 1 \cdot \vec{v}$ for any $\vec{v} \neq \vec{0}$.
so $\boxed{1}$ is the only Eigenvalue of I_V.

2. Which vectors in V are eigenvectors of I_V?

$I_V(\vec{v}) = 1 \cdot \vec{v} \quad \forall \vec{v} \neq \vec{0}$.
so all $\vec{v} \neq \vec{0}$ are eigenvectors of I_V.

3. True or False? Explain your reasoning.

For any linear map $T : V \to V$, if u and v are eigenvectors of T then $u + v$ is also an eigenvector of T.

False. Example 1: if $\vec{u}$ & $\vec{v}$ have different eigenvalues, then

$$\left.\begin{aligned} T\vec{u} &= \lambda_1 \vec{u} \\ T\vec{v} &= \lambda_2 \vec{v} \end{aligned}\right\} \quad T(\vec{u}+\vec{v}) = \lambda_1 \vec{u} + \lambda_2 \vec{v} \neq \lambda(\vec{u}+\vec{v})$$

since $\vec{u}$ & $\vec{v}$ must be L.I. by Thm 5-6.

Example 2: If $\vec{v}$ = E-vector, so is $-\vec{v}$, but
$\vec{v} + -\vec{v} = \vec{0} \neq$ E-vector.