Solution

Math 108A - Quiz # 1

April 17, 2009

1. (a) Define the **span** of $\{v_1, v_2, \ldots, v_n\}$. What do $v_1, \ldots, v_n$ represent in this definition?

$\mathrm{Span}(\vec{v}_1, \ldots, \vec{v}_n) = \{c_1\vec{v}_1 + \cdots + c_n\vec{v}_n \mid c_i \in F \ \forall i\}$.

$\vec{v}_1, \ldots, \vec{v}_n$ represent vectors (or elements in a vector space)

(b) Write the subspace

$$U = \{(a, 0, 0) + (-2b, 0, b) + (0, a, 0) \mid a, b \in F\} \subseteq F^3$$

as the span of a finite set of vectors.

$U = \{a(1,1,0) + b(-2,0,1) \mid a, b \in F\}$

$= \mathrm{span}((1,1,0), (-2,0,1))$

2. (a) Is the set of vectors $\{(0,0,0), (1,0,2), (-1,0,0)\}$ in F^3 linearly independent? Why or why not?

No. $\vec{0} = 1 \cdot (0,0,0) + 0 \cdot (1,0,2) + 0 \cdot (-1,0,0)$

Some linear combination with at least one nonzero coefficient produces the $\vec{0}$-vector.

(b) Let $V = \mathbb{C} = \mathbb{C}^1$ be the vector space of complex numbers over $F = \mathbb{C}$. Are the vectors $z = 1 + i$ and $w = 2 - i$ linearly independent in V? Why or why not?

NO. $w = \underbrace{\left(\frac{2-i}{1+i}\right)}_{\in \mathbb{C}} z$ so w is a scalar multiple of z.